

⑤ 815 600 63-4-1
①

**PERFORMANCE CHARACTERISTICS OF A
SPACED LOOP ANTENNA FORMED BY
TWO ARBITRARILY ORIENTED
PARALLEL LOOPS**

AD No. **407468**

DDC FILE COPY

407468

DDC
RECORDED
JUN 25 1963
TISIA B

Submitted as Task Summary Report Number XIII. Also submitted as an Interim Development Report covering the period 1 January 1963 to 1 April 1963. Prepared for Navy Department, Bureau of Ships, Electronics Division, Contract Nr. NObsr-89167, Project Ser. Nr. SF 0010801, Task 9258, Date of Report, 1 April 1963. Prepared by Douglas N. Travers.

**SOUTHWEST RESEARCH INSTITUTE
SAN ANTONIO, TEXAS**

(4) 7.60

(5) 818 600

6
9-8111

PERFORMANCE CHARACTERISTICS OF A SPACED LOOP ANTENNA FORMED BY TWO ARBITRARILY ORIENTED PARALLEL LOOPS.

9

Submitted as Task Summary Report Number 13. Also submitted as
an Interim Development Report covering the period 1 January
1 April 1963. Prepared for Navy Department, Bureau of Ships,
Electronics Division, Contract No. NObS-89167, Project Ser. Nr.
SF 0010801, Task 9258, Date of Report, 1 April 1963.

16. Pro

17- (15) (11)

(12) 66p. (13) NA
14 NA 18 + 19 NA
20 2 100

10
Prepared by

Douglas N. Travers, Manager
Radio Direction Finding Research

APPROVED:

W. Lyle Donaldson, Director
Electronics and Electrical
Engineering

SOUTHWEST RESEARCH INSTITUTE
SAN ANTONIO, TEXAS

W. Lyle Donaldson
H. L. Donaldson

ABSTRACT

The spaced loop antenna is of interest in HF-DF because of its superior performance relative to the simple loop with respect to bearing accuracy when reradiating objects are nearby. The shipboard site is a prime example. It is therefore of interest to study in some detail other characteristics of the spaced loop particularly those which are inferior to the simple loop, such as sensitivity.

← This report ^{was} ~~has been~~ prepared in order to make available in a single reference a complete listing of the important performance characteristics of the spaced loop antenna. The exact field equations of a general spaced loop are derived and used as a basis for all other performance characteristics. The coaxial spaced loop and coplanar spaced loop are treated as special cases of the general analysis. Any two loop spaced loop operating in the quadrupole mode may be treated as a special case.

Field patterns are derived for near and far fields, for azimuth and elevation planes, and for both vertical and horizontal polarization, for any spaced loop. These results are plotted to show that certain spaced loops have pattern variations as a function of distance to the source. The field equations are then used to compute the radiation resistance, the effective height, effective area, gain, signal-to-noise ratio, noise figure, minimum observable field strength and impedance. Other characteristics including reradiation error reduction, pattern variations with source distance, construction difficulties and pattern distortion sources are *about* reviewed. Although the report begins with ~~very~~ general concepts *concerning* a spaced loop, all important formulas are explained with numerical examples, so ~~that~~ practical conclusions may be drawn.

↑ Interim period activities are summarized in the last section of the report.

TABLE OF CONTENTS

	<u>Page</u>
1. Introduction to the Problem of the Spaced loop	1
2. Types of Spaced Loops	2
3. Equivalence of the Loop Antenna and a Magnetic Dipole	4
4. Mathematical Basis for Derivation of the Field Equations	5
5. The Electric Field Components for the General Spaced Loop	9
6. The Magnetic Field Components for the General Spaced Loop	13
7. The Electric Field Components for the Simple Loop Sense Antenna	17
8. The Magnetic Field Components for the Simple Loop Sense Antenna	20
9. Summary Tables of the Field Equations	22
10. Combinations of Spaced Loop and Simple Loop Antennas	28
11. Effective Height of the Spaced Loop	31
12. Radiation Resistance of the Spaced Loop	34
13. Gain of the Spaced Loop	36
14. Effective Area of the Spaced Loop	38
15. Signal-to-Noise Ratio of the Spaced Loop Antenna	40
16. Minimum Observable Field Strengths	45

TABLE OF CONTENTS (Cont'd)

	<u>Page</u>
17. Impedance of the Spaced Loop Antenna	50
18. Other Spaced Loop Characteristics	52
19. Comparison with Equivalent Factors for the Simple Loop	57
20. Conclusions	59
21. Bibliography of Spaced Loop Reports and Papers	62
22. Activities for the Interim Period	66

LIST OF ILLUSTRATIONS

<u>Figure</u>		<u>Page</u>
1	Common Spaced Loop Types	3
2	Arbitrary Magnetic Dipole	3
3	Azimuth Plane Patterns for the Spaced Loop $\theta_{lp} = 90^\circ$ (Loops Vertical)	27
4	Radiation Resistance vs Frequency for a Typical High Frequency Spaced Loop (Loop Spacing = 2 Meters) (Loop Area = .5 Meters ²)	37
5		43

LIST OF TABLES

<u>Table</u>		<u>Page</u>
5-1		11
7-1		19
9.1	Field Equations for the General Spaced Loop Antenna	23
9.2	Field Equations for the Coaxial Spaced Loop	24
9.3	Field Equations for the Vertical Coplanar Spaced Loop	24
9.4	Field Equations for the Horizontal Coplanar Spaced Loop	24
9.5	Field Equations for the Arbitrarily Oriented Simple Loop Antenna	25
9.6	Field Equations for the Simple Loop Antenna with Axis Along the X Axis	25

LIST OF TABLES (Cont'd)

<u>Table</u>		<u>Page</u>
9.7	Field Equations for the Simple Loop Antenna with Axis Along the Y Axis	25
9.8	Field Equations for the Simple Loop Antenna with Axis Along the Z Axis	25
9.9	Short Electric Dipole (Along Z Axis)	25

1. Introduction to the Problem of the Spaced Loop

Spaced loop radio direction finding antennas have been frequently proposed and occasionally used since the early days of radio. Friis¹ constructed and tested an elaborate coplanar spaced loop prior to 1925. Many coaxial types have been developed since that time²⁻⁶.

In most instances far field patterns have been derived or measured and shown to possess certain desirably characteristics, including improved directivity over a simple loop.

In this report two fundamentally important characteristics of the spaced loop are considered in detail as they affect the designer. These are (1) variations in the field pattern as the source (or point of observation) is moved from the near to the far field region of the antenna, that is, from $r \ll \lambda$ to $r \gg \lambda$, and (2) factors which determine the signal-to-noise ratio of the spaced loop in a practical receiving system.

-
1. Friis, H. T., "A New Directional Receiving System," Proc. IRE, Vol. 13, December 1925, pp. 685-707.
 2. "Instruction Book for Navy Model DAB-3 Radio Direction Finder Equipment," Collins Radio Company, NAVSHIPS 95073, 10 December 1942.
 3. Crampton, C., "A Note on the Application of Spaced Loop High Frequency Direction Finder in H. M. Ships," Report M. 433, Direction Finder Section, Admiralty Signal Establishment, July 1942.
 4. Travers, Douglas N., "A New Shipboard Direction Finding Antenna for the Reduction of Reradiation Error," Southwest Research Institute, 1 September 1960.
 5. Travers, Douglas N., et al. "A Method of Goniometer Scanning the Coaxial Spaced Loop Direction Finder for Vertical Polarization," Task Summary Report VIII, Southwest Research Institute, 1 April 1961.
 6. Evans, G., "The Crossed-Spaced-Loop Direction-Finder Aerial," IRE Transactions on Antennas and Propagation, Vol. AP-10 Number 6, November 1962, pp. 686-691.

It is shown that certain spaced loops have a radiation pattern which is a function of the distance to the source or point of observation. Furthermore multimode arrangements, such as use of a simple loop in conjunction with a spaced loop, always produce a pattern which is a function of distance to the source. As a result analyses based on far field patterns may lead to erroneous conclusions concerning the effect of nearby sources (near field sources less than one wavelength away) such as shipboard reradiators.

In another section of this report, patterns are derived some of which have been experimentally verified.⁴ Criteria for avoiding the various difficulties are given and near field testing procedures outlined. It is concluded that in general one should not attempt to design a spaced loop or multimode loop array for use when both near and far field sources may be of interest (as for instance an HF/DF antenna on a ship, or for testing in a screen room) without due consideration for both near and far field effects. The method of analysis given in this report is adequate for most practical problems involving the common spaced loop direction finding antennas.

The second important characteristic of the spaced loop is the signal-to-noise ratio under practical conditions. The signal-to-noise ratio is derived in terms of the radiation resistance, effective height, loss resistance of the antenna and coupling networks and equivalent noise input resistance of the first amplifier. From a specified signal-to-noise ratio the weakest field which can be intercepted can be shown in terms of the various parameters. It is concluded that at the present state of the art amplifier input noise resistance is the limiting factor, antenna loss resistance is an order of magnitude lower and antenna radiation resistance (and radiation resistance temperature) is quite negligible compared to both. It therefore appears that reduction of amplifier input noise resistance combined with cooling can offer a substantial improvement in signal-to-noise ratio. If sufficient improvement can be attained in this manner, further improvement would require cooling of the antenna loss resistance. The ultimate limit of sensitivity is determined by the temperature of the radiation resistance which is really the temperature of the sources in the vicinity of the antenna. However, at the present state of the art this theoretical limit is at least 60 to 80 db below present practice, hence the need for further theoretical understanding of the factors which limit present practice in the practical case.

2. Types of Spaced Loops

Most spaced loop antennas are variations of the forms shown in Figure 1, with or without additional loop elements such as sense antennas or signal injection antennas. All such spaced loops are variations of a common form which is an arbitrary array of at least two magnetic dipoles.

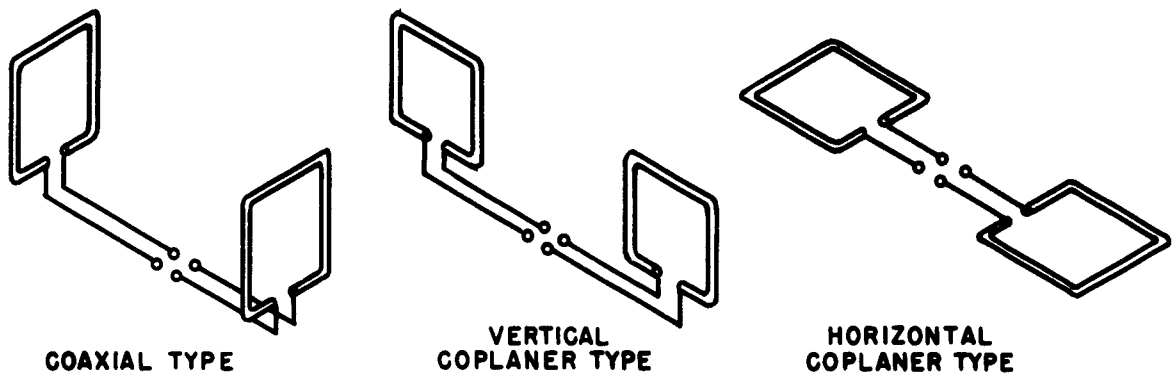


FIGURE 1

COMMON SPACED LOOP TYPES

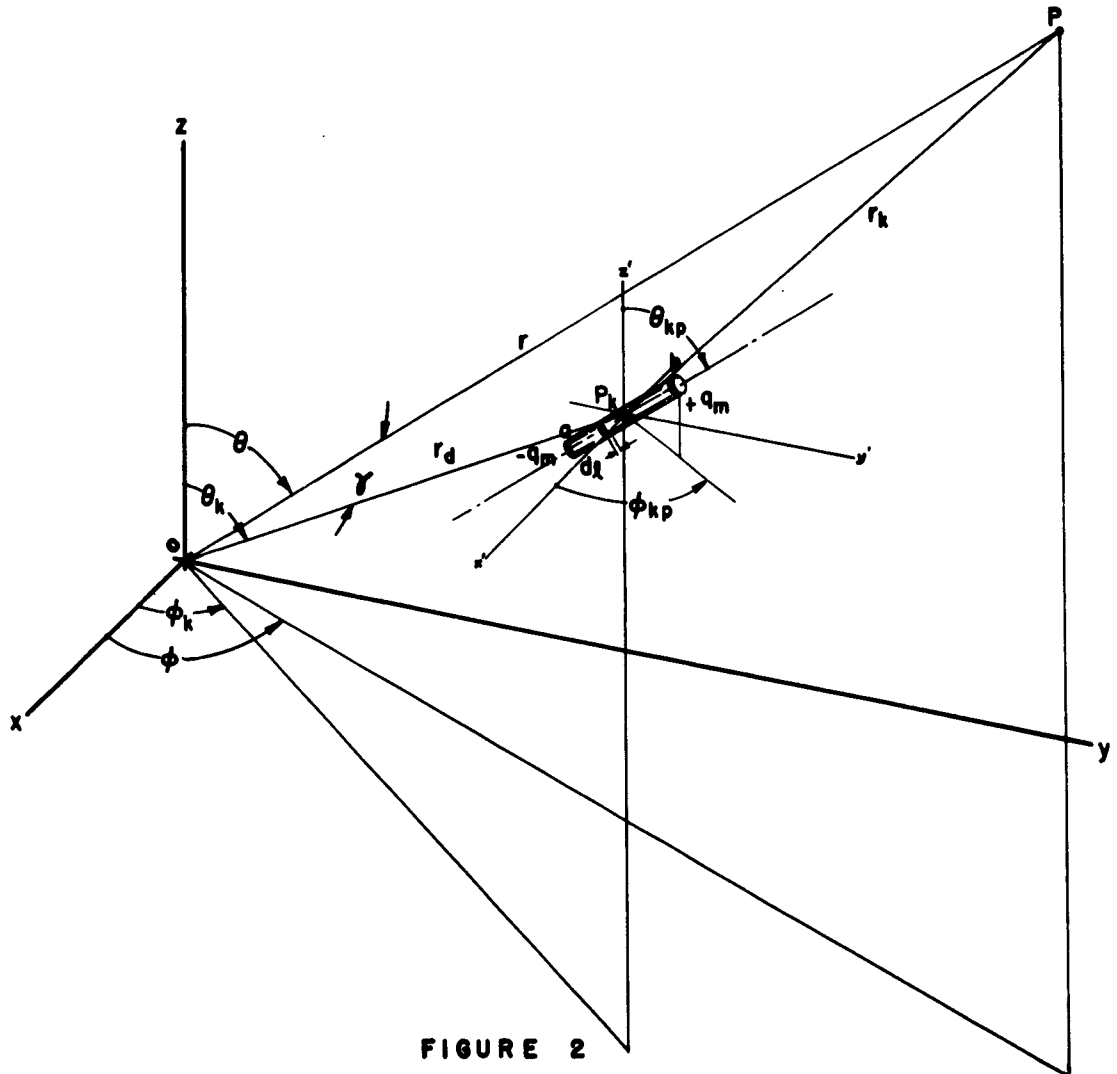


FIGURE 2

ARBITRARY MAGNETIC DIPOLE

Figure 2 shows a single magnetic dipole in an arbitrary location $P_k(x, y, z)$ with arbitrary polarization. When two such dipoles are positioned in an array, all of the familiar two element spaced loops can be formed by appropriate assignment of the various parameters. Certain arrangements of two dipoles will produce quadrupoles. The spaced loop modes of interest in this report are quadrupole modes.

It will be noted that the coaxial and coplanar spaced loops are limiting cases of the general spaced loop. Any intermediate angle for the dipole orientations will also produce the quadrupole mode. These intermediate spaced loops have been of little interest to date because of the lack of complete symmetry.

3. Equivalence of the Loop Antenna and a Magnetic Dipole

Analysis of loop arrays may be simplified by treating the small loop as a short magnetic dipole. Various standard references illustrate this equivalence⁷⁻⁹. Kraus develops the equivalence in a simple manner as follows.

The magnetic dipole is assumed to carry a fictitious magnetic current I_m . The moment of the magnetic dipole is $q_m L$ where q_m is the pole strength at each end of the dipole and L is the dipole length. The magnetic current is related to this pole strength by

$$I_m = -\mu \frac{dq_m}{dt} \quad (1)$$

where

$$I_m = I_{m0} e^{j\omega t}$$

μ = permeability of an isotropic homogeneous medium

-
7. Kraus, J. D., Antennas, McGraw-Hill Book Company, New York, 1959, p. 157.
 8. Stratton, Julius Adams, Electromagnetic Theory, McGraw-Hill Book Company, New York, 1941.
 9. Smythe, William R., Static and Dynamic Electricity, McGraw-Hill Book Company, New York, 1950.

Integrating (1) yields

$$q_m = - \frac{I_m}{j\omega\mu} \quad (2)$$

The magnetic moment of the true loop is IA where A is the loop area and I is the loop current. Equating this to the moment of the magnetic dipole, we have

$$q_m L = IA \quad (3)$$

Substituting (2) into (3)

$$\frac{I_m L}{j\omega\mu} = - IA \quad (4)$$

This may be rearranged as follows

$$I_m L = - j\omega\mu IA \quad (5)$$

The analysis which follows is performed in terms of dipoles equivalent to real loops of area A carrying a current I .

4. Mathematical Basis for Derivation of the Field Equations

The problem to be solved is the determination of the exact field equations of an arbitrary array of magnetic dipoles. A specific solution will be determined for two dipoles arbitrarily located and polarized, arranged to form a generalized quadrupole. The solution for a single dipole acting as a sense antenna will also be given.

By virtue of the equivalence of the magnetic dipole and a real loop, the electric field may be determined from the relation¹⁰:

$$E = \frac{1}{\mu} \nabla \times F \quad (6)$$

where F is the vector potential of the fictitious magnetic current I_m .

This is analogous to the usual procedure of obtaining the magnetic field of an electric dipole array from the vector potential A by means of the relation

$$H = \frac{1}{\mu} \nabla \times A \quad (7)$$

10. Kraus, J. D., Antennas, McGraw-Hill Book Company, New York, 1950, p. 159.

In the present case, for sinusoidal time varying fields, the magnetic components may be determined from the relation

$$H = j \frac{\nabla \times E}{\omega \mu} \quad (8)$$

By analogy to A, F is given by

$$F = \frac{\mu}{4\pi} \int_V \frac{J_m}{r} d\tau \quad (9)$$

For the dipole at P_k of Figure 2, equation (9) may be written as

$$F_k = \frac{\mu}{4\pi} \int_a^b \frac{I_m e^{j\omega} dl}{r_k} \left(t - \frac{r_k}{c} \right) \quad (10)$$

When the dipole is very short compared to r_k , I_m over the dipole is constant, the dipole length is L , and hence equation (10) may be written as

$$F_k = \frac{\mu(I_m L)_k}{4\pi r_k} e^{j\omega} \left(t - \frac{r_k}{c} \right) \quad (11)$$

If F_k is oriented in any arbitrary direction as shown, then the rectangular components of F_k are

$$\begin{aligned} F_{kx} &= F_k \sin \theta_{kp} \cos \phi_{kp} \\ F_{ky} &= F_k \sin \theta_{kp} \sin \phi_{kp} \\ F_{kz} &= F_k \cos \theta_{kp} \end{aligned} \quad (12)$$

These are related to the spherical components as follows

$$\begin{aligned} F_{kr} &= F_{kx} \sin \theta \cos \phi + F_{ky} \sin \theta \sin \phi + F_{kz} \cos \theta \\ F_{k\theta} &= F_{kx} \cos \theta \cos \phi + F_{ky} \cos \theta \sin \phi - F_{kz} \sin \theta \\ F_{k\phi} &= -F_{kx} \sin \phi + F_{ky} \cos \phi \end{aligned} \quad (13)$$

Substituting equations (12) into (13) and simplifying yields

$$\begin{aligned} F_{kr} &= F_k [\sin \theta \sin \theta_{kp} \cos(\phi_{kp} - \phi) + \cos \theta \cos \theta_{kp}] \\ F_{k\theta} &= F_k [\cos \theta \sin \theta_{kp} \cos(\phi_{kp} - \phi) - \sin \theta \cos \theta_{kp}] \\ F_{k\phi} &= F_k [\sin \theta_{kp} \sin(\phi_{kp} - \phi)] \end{aligned} \quad (14)$$

Reference to Figure 2 shows the dipole is located a distance r_k from the point of observation where r_k is given exactly by

$$r_k^2 = r^2 + r_d^2 - 2rr_d \cos \gamma \quad (15)$$

where

$$\cos \gamma = \sin \theta \sin \theta_k \cos(\phi - \phi_k) + \cos \theta \cos \theta_k$$

A binomial expansion of (15) yields

$$r_k = r \left[1 - \frac{r_d}{r} \cos \gamma + \left(\frac{r_d}{r} \right)^2 \left(\frac{1 - \cos^2 \gamma}{2} \right) + \dots \right] \quad (16)$$

When $r \gg r_d$ the above is approximately equal to

$$r_k = r \left[1 - \frac{r_d}{r} \cos \gamma \right] \quad (17)$$

Substituting equation (17) into (11) yields

$$F_k = \frac{\mu(I_m L)_k e^{j\omega \left(t - \frac{r - r_d \cos \gamma}{c} \right)}}{4\pi r \left[1 - \frac{r_d}{r} \cos \gamma \right]} \quad (18)$$

A spaced loop will consist of two dipoles located colinear with the origin and on the surface of a sphere of radius r_d . This requires that

$$\gamma_1 + \gamma_2 = \pi \quad (19)$$

Substituting equation (19) into (18) to obtain retarded potentials for each dipole yields:

$$F_1 = \frac{\mu(I_m L)_1 e^{j\omega \left[t - \left(\frac{r - r_d \cos \gamma}{c} \right) \right]}}{4\pi r \left[1 - \frac{r_d}{r} \cos \gamma_1 \right]} \quad (20)$$

$$F_2 = \frac{\mu(I_m L)_2 e^{j\omega \left[t - \left(\frac{r + r_d \cos \gamma}{c} \right) \right]}}{4\pi r \left[1 + \frac{r_d}{r} \cos \gamma_1 \right]}$$

where

$$\cos \gamma_1 = \sin \theta \sin \theta_1 \cos(\phi - \phi_1) + \cos \theta \cos \theta_1 \quad (21)$$

An orientation which is convenient for interpretation of the field equations places the dipoles on either the x or y axis. This permits the E_θ component to represent vertical polarization and the E_ϕ component, horizontal polarization (although the effect of the earth has not been considered in this analysis). Choosing the y axis requires that $\theta_1 = \phi_1 = 90^\circ$ and consequently

$$\cos \gamma_1 = \sin \theta \sin \phi \quad (22)$$

Rearranging equations (20) to have a common denominator and neglecting terms $(r_d/r)^2$ and higher order we have

$$F_1 = \frac{\mu(I_m L)_1 e^{j\omega \left[t - \left(\frac{r - r_d \sin \theta \sin \phi}{c} \right) \right]}}{4\pi r} \left[1 + \frac{r_d}{r} \sin \theta \sin \phi \right] \quad (23)$$

$$F_2 = \frac{\mu(I_m L)_2 e^{j\omega \left[t - \left(\frac{r + r_d \sin \theta \sin \phi}{c} \right) \right]}}{4\pi r} \left[1 - \frac{r_d}{r} \sin \theta \sin \phi \right] \quad (24)$$

The retarded potential of the quadrupole mode is

$$F_1 - F_2 = - \frac{\mu(I_m L) e^{j\omega \left(t - \frac{r}{c} \right)} \beta^2 r_d \sin \theta \sin \phi}{2\pi} \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} \right] \quad (25)$$

where $(I_m L)_1 = (I_m L)_2$ and the dipoles are arranged to be parallel. Parallel dipoles require that

$$\theta_{1p} + \theta_{2p} = \pi \quad (26)$$

$$\phi_{2p} = \phi_{1p} + \pi$$

Equations (14) then become

$$\begin{aligned} F_r &= [F_1 - F_2] [\sin \theta \sin \theta_{1p} \cos(\phi_{1p} - \phi) + \cos \theta \cos \theta_{1p}] \\ F_\theta &= [F_1 - F_2] [\cos \theta \sin \theta_{1p} \cos(\phi_{1p} - \phi) - \sin \theta \cos \theta_{1p}] \\ F_\phi &= [F_1 - F_2] [\sin \theta_{1p} \sin(\phi_{1p} - \phi)] \end{aligned} \quad (27)$$

5. The Electric Field Components for the General Spaced Loop

Equations (27) may be substituted into equation (6) to obtain the electric field equations. Expanding equation (6) yields

$$\begin{aligned} \mu E_r &= \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta F_\phi) - \frac{\partial F_\theta}{\partial \phi} \right] = \frac{1}{r \sin \theta} \left[\sin \theta \frac{\partial F_\phi}{\partial \theta} + \right. \\ &\quad \left. + \cos \theta F_\phi - \frac{\partial F_\theta}{\partial \phi} \right] \end{aligned} \quad (28)$$

$$\mu E_\theta = \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r} (r F_\phi) \right] = \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial F_r}{\partial \phi} - r \frac{\partial F_\phi}{\partial r} - F_\phi \right] \quad (29)$$

$$\mu E_\phi = \frac{1}{r} \left[\frac{\partial}{\partial r} (r F_\theta) - \frac{\partial F_r}{\partial \theta} \right] = \frac{1}{r} \left[r \frac{\partial F_\theta}{\partial r} + F_\theta - \frac{\partial F_r}{\partial \theta} \right] \quad (30)$$

The six required derivatives are

$$\begin{aligned} \frac{\partial F_r}{\partial \theta} &= - \frac{\mu(I_m L)_1 \beta^2 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} \right] \sin \phi \left[\sin 2\theta \sin \theta_{1p} \cos(\phi_{1p} - \phi) + \right. \\ &\quad \left. + \cos 2\theta \cos \theta_{1p} \right] \end{aligned} \quad (31)$$

$$\begin{aligned} \frac{\partial F_r}{\partial \phi} &= - \frac{\mu(I_m L)_1 \beta^2 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left[\frac{1}{(j\beta r)} + \frac{1}{(j\beta r)^2} \right] \sin \theta \left[\sin \theta \sin \theta_{1p} \cos(\phi_{1p} - 2\phi) + \right. \\ &\quad \left. + \cos \theta \cos \theta_{1p} \cos \phi \right] \end{aligned} \quad (32)$$

$$\begin{aligned} \frac{\partial F_\theta}{\partial r} = & + \frac{j\mu(I_m L)_1 \beta^3 r_{de}}{2\pi} e^{j\omega\left(t - \frac{r}{c}\right)} \sin\theta \sin\phi \left[\frac{1}{j\beta r} + \frac{2}{(j\beta r)^2} + \frac{2}{(j\beta r)^3} \right] \times \\ & \times [\cos\theta \sin\theta_{1p} \cos(\phi_{1p} - \phi) - \sin\theta \cos\theta_{1p}] \end{aligned} \quad (33)$$

$$\begin{aligned} \frac{\partial F_\theta}{\partial \phi} = & - \frac{\mu(I_m L)_1 \beta^2 r_{de}}{2\pi} e^{j\omega\left(t - \frac{r}{c}\right)} \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} \right] \sin\theta [\cos\theta \sin\theta_{1p} \cos(\phi_{1p} - 2\phi) \\ & - \sin\theta \cos\theta_{1p} \cos\phi] \end{aligned} \quad (34)$$

$$\begin{aligned} \frac{\partial F_\phi}{\partial r} = & + \frac{j\mu(I_m L)_1 \beta^3 r_{de}}{2\pi} e^{j\omega\left(t - \frac{r}{c}\right)} \left[\frac{1}{j\beta r} + \frac{2}{(j\beta r)^2} + \frac{2}{(j\beta r)^3} \right] \times \\ & \times \sin\theta \sin\theta_{1p} \sin\phi \sin(\phi_{1p} - \phi) \end{aligned} \quad (35)$$

$$\begin{aligned} \frac{\partial F_\phi}{\partial \theta} = & - \frac{\mu(I_m L)_1 \beta^2 r_{de}}{2\pi} e^{j\omega\left(t - \frac{r}{c}\right)} \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} \right] \cos\theta \sin\theta_{1p} \sin\phi \sin(\phi_{1p} - \phi) \end{aligned} \quad (36)$$

Substituting these derivatives and equations (25) and (27) into equations (28) thru (30) yields the electric field equations

$$\begin{aligned} E_r = & -j \frac{(I_m L)_1 \beta^3 r_{de}}{2\pi} e^{j\omega\left(t - \frac{r}{c}\right)} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] [\sin\theta \sin\theta_{1p} \cos\phi \\ & - \cos\theta \sin\theta_{1p} \cos\phi_{1p}] \end{aligned} \quad (37)$$

$$\begin{aligned} E_\theta = & -j \frac{(I_m L)_1 \beta^3 r_{de}}{2\pi} e^{j\omega\left(t - \frac{r}{c}\right)} \left\{ \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] [\sin\theta \sin\theta_{1p} \times \right. \\ & \times [\cos\phi \cos(\phi_{1p} - \phi) + 2 \sin\phi \sin(\phi_{1p} - \phi)] + \cos\theta \cos\theta_{1p} \cos\phi] + \\ & \left. + \left[\frac{1}{j\beta r} \right] \sin\theta \sin\theta_{1p} \sin\phi \sin(\phi_{1p} - \phi) \right\} \end{aligned} \quad (38)$$

$$E_{\phi} = j \frac{(I_m L)_1 \beta^3 r_d e^{j\omega \left(t - \frac{r}{c}\right)}}{2\pi} \left\{ \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] [3 \sin \theta \cos \theta \sin \theta_{1p} \cos(\phi_{1p} - \phi) + (1 - 3 \sin^2 \theta) \cos \theta_{1p}] \sin \phi + \left[\frac{1}{j\beta r} \right] \sin \theta \sin \phi [\cos \theta \sin \theta_{1p} \cos(\phi_{1p} - \phi) - \sin \theta \cos \theta_{1p}] \right\} \quad (39)$$

where

$$(I_m L)_1 = -j\omega\mu I_1 A_1 \quad (5)$$

Referring to equations (38) and (39), the criteria for equal patterns in near and far field regions can be found by setting the appropriate terms equal. For E_{θ} this requires that

$$\sin \theta \sin \theta_{1p} [\cos \phi \cos(\phi_{1p} - \phi) + 2 \sin \phi \sin(\phi_{1p} - \phi)] + \cos \theta \cos \theta_{1p} \cos \phi = a_0 \sin \theta \sin \theta_{1p} \sin \phi \sin(\phi_{1p} - \phi) \quad (40)$$

where a_0 is arbitrary. This requires that $\theta_{1p} = 90^\circ$ and $\phi_{1p} = 90^\circ$ or 270° so that $3 \sin \theta \sin \phi \cos \phi = a_0 \sin \theta \sin \phi \cos \phi$, or $a_0 = 3$. This appears to be the only set of values for θ_{1p} and ϕ_{1p} which produce equal far and near field patterns. These values correspond to the coaxial spaced loop (shown first in Figure 1). A similar result is obtained for the E_{ϕ} component.

It is evident that, in general, the antenna patterns are not the same in both near and far field regions even though the antenna dimensions are small compared to the wavelength.

The three principal spaced loops are the coaxial, the vertical coplanar usually called simply "coplanar," and the horizontal coplanar. The polarization parameters for these are as follows:

TABLE 5-1

Type	θ_{1p}	ϕ_{1p}
Coaxial	90°	90°
Vertical Coplanar	90°	0
Horizontal Coplanar	0	N. A.

Substitution of these values and equation (5) into (37), (38) and (39) yields the following:

a. The Coaxial Spaced Loop

$$E_r = 0 \quad (41)$$

$$E_\theta = - \frac{I\beta^3 A r_d \omega \mu e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{3}{(j\beta r)^3} + \frac{3}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \theta \sin 2\phi \quad (42)$$

$$E_\phi = \frac{I\beta^3 A r_d \omega \mu e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{3}{(j\beta r)^3} + \frac{3}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin 2\theta \sin^2 \phi \quad (43)$$

b. The Vertical Coplanar Spaced Loop

$$E_r = \frac{I\beta^3 A r_d \omega \mu e^{j\omega \left(t - \frac{r}{c}\right)}}{2\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \cos \theta \quad (44)$$

$$E_\theta = - \frac{I\beta^3 A r_d \omega \mu e^{j\omega \left(t - \frac{r}{c}\right)}}{2\pi} \left\{ \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] (\cos 2\phi - \sin^2 \phi) - \frac{\sin^2 \phi}{j\beta r} \right\} \sin \theta \quad (45)$$

$$E_\phi = \frac{I\beta^3 A r_d \omega \mu e^{j\omega \left(t - \frac{r}{c}\right)}}{8\pi} \left[\frac{3}{(j\beta r)^2} + \frac{3}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin 2\theta \sin 2\phi \quad (46)$$

c. The Horizontal Coplanar Spaced Loop

$$E_r = - \frac{I\beta^3 A r_d \omega \mu e^{j\omega \left(t - \frac{r}{c}\right)}}{2\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \sin \theta \cos \phi \quad (47)$$

$$E_{\theta} = -\frac{I\beta^3 A_{rd}\omega\mu e}{2\pi} \frac{j\omega \left(t - \frac{r}{c}\right)}{\left[(j\beta r)^3 + \frac{1}{(j\beta r)^2}\right]} \cos \theta \cos \phi \quad (48)$$

$$E_{\phi} = \frac{I\beta^3 A_{rd}e}{2\pi} \frac{j\omega \left(t - \frac{r}{c}\right)}{\left[(j\beta r)^3 + \frac{1}{(j\beta r)^2}\right]} \left\{ \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] (\cos 2\theta - \sin^2 \theta) - \frac{\sin^2 \theta}{j\beta r} \right\} \sin \phi \quad (49)$$

The above relations show that two special cases produce E field patterns which differ for near and far field sources. These are: (1) azimuth or ϕ plane patterns for the vertical coplanar spaced loop for vertical polarization, and (2) elevation or θ plane patterns for the horizontal coplanar spaced loop for horizontal polarization.

All other E field patterns for the above three special cases maintain a constant shape for sources at any distance which is large compared to the spaced loop dimensions. It is to be emphasized that the cases where the patterns differ with source distance do not result from a parallax effect. The pattern shape changes due to parallax have not been calculated (they have been assumed to be negligible).

6. The Magnetic Field Components for the General Spaced Loop

As described in section 4 the magnetic field equations are easily obtained from Curl E by equation (8). Expanding equation (8) yields

$$H_r = \frac{j}{\omega\mu r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta E_{\phi}) - \frac{\partial E_{\theta}}{\partial \phi} \right] \quad (50)$$

$$H_{\theta} = \frac{j}{\omega\mu} \left[\frac{1}{r \sin \theta} \frac{\partial E_r}{\partial \phi} - \frac{1}{r} \frac{\partial}{\partial r} (r E_{\phi}) \right] \quad (51)$$

$$H_{\phi} = \frac{j}{\omega\mu r} \left[\frac{\partial}{\partial r} (r E_{\theta}) - \frac{\partial E_r}{\partial \theta} \right] \quad (52)$$

These may be rewritten as follows

$$H_r = \frac{j}{\omega\mu r} \left[\frac{\partial E_{\phi}}{\partial \theta} + \frac{E_{\phi} \cos \theta}{\sin \theta} - \frac{\partial E_{\theta}}{\sin \theta \partial \phi} \right] \quad (53)$$

$$H_\theta = \frac{j}{\omega\mu} \left[\frac{1}{r \sin \theta} \frac{\partial E_r}{\partial \phi} - \frac{E_\phi}{r} - \frac{\partial E_\phi}{\partial r} \right] \quad (54)$$

$$H_\phi = \frac{j}{\omega\mu} \left[\frac{\partial E_\theta}{\partial r} + \frac{E_\theta}{r} - \frac{\partial E_r}{r \partial \theta} \right] \quad (55)$$

The six derivatives required are as follows

$$\begin{aligned} \frac{\partial E_r}{\partial \theta} = & -j \frac{(I_m L) \beta^3 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] (\cos \theta \cos \theta_{1p} \cos \phi + \\ & + \sin \theta \sin \theta_{1p} \cos \phi_{1p}) \end{aligned} \quad (56)$$

$$\frac{\partial E_r}{\partial \phi} = +j \frac{(I_m L) \beta^3 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \sin \theta \cos \theta_{1p} \sin \phi \quad (57)$$

$$\begin{aligned} \frac{\partial E_\theta}{\partial r} = & - \frac{(I_m L) \beta^4 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left\{ \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \times \right. \\ & \times \left[\sin \theta \sin \theta_{1p} [\cos \phi \cos (\phi_{1p} - \phi) + 2 \sin \phi \sin (\phi_{1p} - \phi)] + \cos \theta \cos \theta_{1p} \cos \phi \right] \\ & \left. + \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \theta \sin \theta_{1p} \sin \phi \sin (\phi_{1p} - \phi) \right\} \end{aligned} \quad (58)$$

$$\begin{aligned} \frac{\partial E_\theta}{\partial \phi} = & -j \frac{(I_m L) \beta^3 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left\{ \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \times \right. \\ & \times 3 [\sin \theta \sin \theta_{1p} \sin (\phi_{1p} - 2\phi) - \cos \theta \cos \theta_{1p} \sin \phi] + \\ & \left. + \left[\frac{1}{j\beta r} \right] \sin \theta \sin \theta_{1p} \sin (\phi_{1p} - 2\phi) \right\} \end{aligned} \quad (59)$$

$$\begin{aligned} \frac{\partial E_\phi}{\partial r} = & \frac{(I_m L) \beta^4 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left\{ \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \times \right. \\ & \times [3 \sin \theta \cos \theta \sin \theta_{1p} \cos (\phi_{1p} - \phi) + (1 - 3 \sin^2 \theta) \cos \theta_{1p}] \sin \phi + \\ & \left. + \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \theta \sin \phi [\cos \theta \sin \theta_{1p} \cos (\phi_{1p} - \phi) - \sin \theta \cos \theta_{1p}] \right\} \end{aligned} \quad (60)$$

$$H_\theta = \frac{j}{\omega\mu} \left[\frac{1}{r \sin \theta} \frac{\partial E_r}{\partial \phi} - \frac{E_\phi}{r} - \frac{\partial E_\phi}{\partial r} \right] \quad (54)$$

$$H_\phi = \frac{j}{\omega\mu} \left[\frac{\partial E_\theta}{\partial r} + \frac{E_\theta}{r} - \frac{\partial E_r}{r \partial \theta} \right] \quad (55)$$

The six derivatives required are as follows

$$\begin{aligned} \frac{\partial E_r}{\partial \theta} = -j \frac{(I_m L) \beta^3 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] (\cos \theta \cos \theta_{1p} \cos \phi + \\ + \sin \theta \sin \theta_{1p} \cos \phi_{1p}) \end{aligned} \quad (56)$$

$$\frac{\partial E_r}{\partial \phi} = +j \frac{(I_m L) \beta^3 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \sin \theta \cos \theta_{1p} \sin \phi \quad (57)$$

$$\begin{aligned} \frac{\partial E_\theta}{\partial r} = - \frac{(I_m L) \beta^4 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left\{ \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \times \right. \\ \times \left[\sin \theta \sin \theta_{1p} [\cos \phi \cos(\phi_{1p} - \phi) + 2 \sin \phi \sin(\phi_{1p} - \phi)] + \cos \theta \cos \theta_{1p} \cos \phi \right] \\ \left. + \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \theta \sin \theta_{1p} \sin \phi \sin(\phi_{1p} - \phi) \right\} \end{aligned} \quad (58)$$

$$\begin{aligned} \frac{\partial E_\theta}{\partial \phi} = -j \frac{(I_m L) \beta^3 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left\{ \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \times \right. \\ \times 3 [\sin \theta \sin \theta_{1p} \sin(\phi_{1p} - 2\phi) - \cos \theta \cos \theta_{1p} \sin \phi] + \\ \left. + \left[\frac{1}{j\beta r} \right] \sin \theta \sin \theta_{1p} \sin(\phi_{1p} - 2\phi) \right\} \end{aligned} \quad (59)$$

$$\begin{aligned} \frac{\partial E_\phi}{\partial r} = \frac{(I_m L) \beta^4 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left\{ \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \times \right. \\ \times [3 \sin \theta \cos \theta \sin \theta_{1p} \cos(\phi_{1p} - \phi) + (1 - 3 \sin^2 \theta) \cos \theta_{1p}] \sin \phi + \\ \left. + \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \theta \sin \phi [\cos \theta \sin \theta_{1p} \cos(\phi_{1p} - \phi) - \sin \theta \cos \theta_{1p}] \right\} \end{aligned} \quad (60)$$

$$\frac{\partial E_\phi}{\partial \theta} = j \frac{(I_m L) \beta^3 r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left\{ \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \times \right. \\ \times [3 \cos 2\theta \sin \theta_{1p} \cos(\phi_{1p} - \phi) - 3 \sin 2\theta \cos \theta_{1p}] \sin \phi + \\ \left. + \left[\frac{1}{j\beta r} \right] [\cos 2\theta \sin \theta_{1p} \sin \phi \cos(\phi_{1p} - \phi) - \sin 2\theta \cos \theta_{1p} \sin \phi] \right\} \quad (61)$$

Substituting these derivatives into (53), (54) and (55) with equations (38) and (39) yields

$$H_r = \frac{-I\beta^4 A r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{4\pi} \left\{ \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \left[\sin \phi [2 \sin \theta_{1p} (\cos 2\theta + \right. \right. \\ \left. \left. + \cos^2 \theta) \cos(\phi_{1p} - \phi) - 3 \sin 2\theta \cos \theta_{1p}] + \sin \theta_{1p} \sin(\phi_{1p} - 2\phi) \right] \right\} \quad (62)$$

$$H_\theta = \frac{-I\beta^4 A r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left\{ \left[\frac{1}{(j\beta r)^4} + \frac{1}{(j\beta r)^3} \right] \cos \theta_{1p} \sin \phi + \left[\frac{2}{(j\beta r)^4} + \frac{2}{(j\beta r)^3} + \right. \right. \\ \left. \left. + \frac{1}{(j\beta r)^2} \right] \sin \phi [3 \sin \theta \cos \theta \sin \theta_{1p} \cos(\phi_{1p} - \phi) + (1 - 3 \sin^2 \theta) \cos \theta_{1p}] + \right. \\ \left. + \left[\frac{1}{j\beta r} \right] \sin \phi \sin \theta [\cos \theta \sin \theta_{1p} \cos(\phi_{1p} - \phi) - \sin \theta \cos \theta_{1p}] \right\} \quad (63)$$

$$H_\phi = \frac{-I\beta^4 A r_d e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left\{ \left[\frac{1}{(j\beta r)^4} + \frac{1}{(j\beta r)^3} \right] [\cos \theta \cos \theta_{1p} \cos \phi + \right. \\ \left. + \sin \theta \sin \theta_{1p} \cos \phi_{1p}] + \left[\frac{2}{(j\beta r)^4} + \frac{2}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \left[\sin \theta \sin \theta_{1p} \times \right. \\ \left. \times [\cos \phi \cos(\phi_{1p} - \phi) + 2 \sin \phi \sin(\phi_{1p} - \phi)] + \cos \theta \cos \theta_{1p} \cos \phi \right] + \\ \left. + \left[\frac{1}{j\beta r} \right] \times \sin \theta \sin \theta_{1p} \sin \phi \sin(\phi_{1p} - \phi) \right\} \quad (64)$$

Substituting appropriate values from Table 5-1 equations (62) through (64) yield:

a. The Coaxial Spaced Loop

$$H_r = \frac{-I\beta^4 r_d e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] 2(1 - 3 \sin^2 \theta \sin^2 \phi) \quad (65)$$

$$H_\theta = \frac{-I\beta^4 r_d e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{6}{(j\beta r)^4} + \frac{6}{(j\beta r)^3} + \frac{3}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin 2\theta \sin^2 \phi \quad (66)$$

$$H_\phi = \frac{-I\beta^4 A_r d e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{6}{(j\beta r)^4} + \frac{6}{(j\beta r)^3} + \frac{3}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \theta \sin 2\phi \quad (67)$$

b. The Vertical Coplanar Spaced Loop

$$H_r = \frac{I\beta^4 A_r d e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] 3 \sin^2 \theta \sin 2\phi \quad (68)$$

$$H_\theta = \frac{-I\beta^4 A_r d e^{j\omega \left(t - \frac{r}{c}\right)}}{8\pi} \left[\frac{6}{(j\beta r)^4} + \frac{6}{(j\beta r)^3} + \frac{3}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin 2\theta \sin 2\phi \quad (69)$$

$$H_{\phi} = \frac{-I\beta^4 A r_d e^{j\omega \left(t - \frac{r}{c}\right)}}{2\pi} \left\{ \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} \right] \cos 2\phi + \left[\frac{1}{(j\beta r)^2} \right] (\cos 2\phi - \sin^2 \phi) - \frac{\sin^2 \phi}{j\beta r} \right\} \quad (70)$$

c. The Horizontal Coplanar Spaced Loop

$$H_r = \frac{I\beta^4 A r_d e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] 3 \sin 2\theta \sin \phi \quad (71)$$

$$H_{\theta} = \frac{-I\beta^4 A r_d e^{j\omega \left(t - \frac{r}{c}\right)}}{2\pi} \left\{ \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} \right] \cos 2\theta + \left[\frac{1}{(j\beta r)^2} \right] (\cos 2\theta - \sin^2 \theta) - \frac{\sin^2 \theta}{j\beta r} \right\} \sin \phi \quad (72)$$

$$H_{\phi} = \frac{-I\beta^4 A r_d e^{j\omega \left(t - \frac{r}{c}\right)}}{2\pi} \left\{ \frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right\} \cos \theta \cos \phi \quad (73)$$

These equations also show near to far field differences for the same cases where differences were found with the electric field equations. These differences were listed at the end of Section 5.

7. The Electric Field Components for the Simple Loop Sense Antenna

A sense dipole located at the center of the spaced loop will have the vector potential given by equation (10) or equation (11) with the added condition that $r_k = r$. Therefore, by substitution into equations (14), the components of F for the sense dipole are.

$$F_r = F[\sin \theta \sin \theta_p \cos(\phi_p - \phi) + \cos \theta \cos \theta_p] \quad (74)$$

$$F_\theta = F[\cos\theta \sin\theta_p \cos(\phi_p - \phi) - \sin\theta \cos\theta_p] \quad (75)$$

$$F_\phi = F[\sin\theta_p \sin(\phi_p - \phi)] \quad (76)$$

where

$$F = \frac{\mu(I_m L) e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi r} \quad (77)$$

The six required derivatives are:

$$\frac{\partial F_r}{\partial \theta} = \frac{j\mu\beta(I_m L) 3e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{1}{j\beta r} \right] [\cos\theta \sin\theta_p \cos(\phi_p - \phi) - \sin\theta \cos\theta_p] \quad (78)$$

$$\frac{\partial F_r}{\partial \phi} = \frac{j\mu\beta(I_m L) 3e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{1}{j\beta r} \right] [\sin\theta \sin\theta_p \sin(\phi_p - \phi)] \quad (79)$$

$$\begin{aligned} \frac{\partial F_\theta}{\partial r} &= \frac{\mu\beta^2(I_m L) 3e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \times \\ &\times [\cos\theta \sin\theta_p \cos(\phi_p - \phi) - \sin\theta \cos\theta_p] \end{aligned} \quad (80)$$

$$\frac{\partial F_\theta}{\partial \phi} = \frac{j\mu\beta(I_m L) 3e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{1}{j\beta r} \right] [\cos\theta \sin\theta_p \sin(\phi_p - \phi)] \quad (81)$$

$$\frac{\partial F_\phi}{\partial r} = \frac{\mu\beta^2(I_m L) 3e^{j\omega \left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin\theta_p \sin(\phi_p - \phi) \quad (82)$$

$$\frac{\partial F_\phi}{\partial \theta} = 0 \quad (83)$$

Substituting equations (78) through (83) and equations (75) and (76) into equations (28) through (30) yields the electric field equations for the sense dipole:

$$E_r = 0 \quad (84)$$

$$E_\theta = \frac{-\beta^2(I_m L)_3 e^{j\omega\left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin\theta_p \sin(\phi_p - \phi) \quad (85)$$

$$E_\phi = \frac{\beta^2(I_m L)_3 e^{j\omega\left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] [\cos\theta \sin\theta_p \cos(\phi_p - \phi) - \sin\theta \cos\theta_p] \quad (86)$$

The sense dipole has a pattern which does not change shape as a function of distance from the origin. Three orientations of the sense dipole are important: along each of the three coordinate axes. The polarization parameters for these arrangements are as given below in Table 7-1.

TABLE 7-1

Dipole Parallel to	{	Type	θ_p	ϕ_p
		X Axis	90°	0
		Y Axis	90°	90°
		Z Axis	0	N. A.

Thus, the field equations for these three cases are:

a. Dipole Parallel to X Axis

$$E_r = 0 \quad (87)$$

$$E_\theta = \frac{\beta^2(I_m L)_3 e^{j\omega\left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin\phi \quad (88)$$

$$E_{\phi} = \frac{\beta^2(I_m L)_3 e^{j\omega \left(t - \frac{r}{c} \right)}}{4\pi} \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \cos \phi \cos \theta \quad (89)$$

b. Dipole Parallel to Y Axis

$$E_r = 0 \quad (90)$$

$$E_{\theta} = \frac{-\beta^2(I_m L)_3 e^{j\omega \left(t - \frac{r}{c} \right)}}{4\pi} \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \cos \phi \quad (91)$$

$$E_{\phi} = \frac{\beta^2(I_m L)_3 e^{j\omega \left(t - \frac{r}{c} \right)}}{4\pi} \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \phi \cos \theta \quad (92)$$

c. Dipole Parallel to Z Axis

$$E_r = 0 \quad (92)$$

$$E_{\theta} = 0 \quad (93)$$

$$E_{\phi} = \frac{-\beta^2(I_m L)_3 e^{j\omega \left(t - \frac{r}{c} \right)}}{4\pi} \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \theta \quad (95)$$

where

$$(I_m L)_3 = -j\omega \mu I_3 A_3 \quad (5)$$

8. The Magnetic Field Components for the Simple Loop Sense Antenna

By combining equations (5) and (84) through (86) with (53) through (55), the magnetic field components are found as follows:

$$H_r = \frac{-j\beta^3 I A e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] [\sin \theta \sin \theta_p \cos(\phi_p - \phi) + \cos \theta \cos \theta_p] \quad (96)$$

$$H_{\theta} = \frac{+j\beta^3 I A e^{j\omega \left(t - \frac{r}{c} \right)}}{4\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] [\cos \theta \sin \theta_p \cos(\phi_p - \phi) - \sin \theta \cos \theta_p] \quad (97)$$

$$H_{\phi} = \frac{j\beta^3 I A e^{j\omega \left(t - \frac{r}{c} \right)}}{4\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \theta_p \sin(\phi_p - \phi) \quad (98)$$

The field equations for each of the three orientations listed in Table 7-1 are as follows:

a. Dipole Parallel to X Axis

$$H_r = \frac{-j\beta^3 I A e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \sin \theta \cos \phi \quad (99)$$

$$H_{\theta} = \frac{j\beta^3 I A e^{j\omega \left(t - \frac{r}{c} \right)}}{4\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \cos \theta \cos \phi \quad (100)$$

$$H_{\phi} = \frac{-j\beta^3 I A e^{j\omega \left(t - \frac{r}{c} \right)}}{4\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \phi \quad (101)$$

b. Dipole Parallel to Y Axis

$$H_r = \frac{-j\beta^3 I A e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \sin \theta \sin \phi \quad (102)$$

$$H_{\theta} = \frac{j\beta^3 I A e^{j\omega \left(t - \frac{r}{c} \right)}}{4\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \cos \theta \sin \phi \quad (103)$$

$$H_{\phi} = \frac{j\beta^3 I A e^{j\omega \left(t - \frac{r}{c} \right)}}{4\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \cos \phi \quad (104)$$

c. Dipole Parallel to Z Axis

$$H_r = \frac{-j\beta^3 I A e^{j\omega\left(t - \frac{r}{c}\right)}}{2\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \cos \theta \quad (105)$$

$$H_\theta = \frac{-j\beta^3 I A e^{j\omega\left(t - \frac{r}{c}\right)}}{4\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \theta \quad (106)$$

$$H_\phi = 0 \quad (107)$$

9. Summary Tables of the Field Equations

The following tables are presented for quick reference and ease of comparison. Plots of the various field patterns are given at the end of this section.

The first four tables refer to the spaced loop with the dipoles (loops) located at points on the y axis ($y = r_d$, $y = -r_d$). Tables 9.5 through 9.8 refer to a simple loop oriented as indicated and located at the origin as would be the case for a sense loop antenna. Table 9.9 refers to a short electric dipole located at the origin and parallel to the z axis as would be the case for an omnidirectional sense antenna of the type for vertical polarization. Table 9.9 is provided from Kraus¹ and is for the purpose of comparison to the other antennas since this type sense antenna is frequently used with simple loop D/F antennas. In all tables the equations are regrouped according to polarization of the field.

Table 9.1	Field equations for the general spaced loop antenna.
Table 9.2	Field equations for the coaxial spaced loop antenna.
Table 9.3	Field equations for the vertical coplanar spaced loop antenna.
Table 9.4	Field equations for the horizontal coplanar spaced loop.
Table 9.5	Field equations for the arbitrarily oriented simple loop antenna.
Table 9.6	Field equations for the simple loop antenna with axis along the X axis.
Table 9.7	Field equations for the simple loop antenna with axis along the Y axis.
Table 9.8	Field equations for the simple loop antenna with axis along the Z axis.
Table 9.9	Field equations for the short electric dipole sense antenna along the Z axis.

TABLE 9-1. FIELD EQUATIONS FOR THE GENERAL SPACED LOOP ANTENNA

	(137)
$E_z = \frac{-j\omega^2 \mu_0 \epsilon_0^2}{2\pi} \left(1 - \frac{z}{c}\right) \left[\frac{1}{(j\omega r)^3} + \frac{1}{(j\omega r)^2} \right] (\sin \theta \cos \theta_{1p} \cos \phi - \cos \theta \sin \theta_{1p} \cos \phi_{1p})$	
	(138)
$E_\theta = \frac{-j\omega^2 \mu_0 \epsilon_0^2}{2\pi} \left(1 - \frac{z}{c}\right) \left[\frac{3}{(j\omega r)^4} + \frac{3}{(j\omega r)^3} + \frac{1}{(j\omega r)^2} \right] \left\{ \sin \phi \left[2 \sin \theta_{1p} (\cos 2\theta + \cos 2\phi) \cos \phi_{1p} - \theta - 3 \sin 2\theta \cos \theta_{1p} \right] + \sin \theta_{1p} \sin \phi_{1p} - 2\theta \right\}$	
	(139)
$E_\phi = \frac{-j\omega^2 \mu_0 \epsilon_0^2}{2\pi} \left(1 - \frac{z}{c}\right) \left\{ \left[\frac{1}{(j\omega r)^3} + \frac{1}{(j\omega r)^2} \right] \left[\sin \theta \sin \theta_{1p} (\cos \phi \cos \phi_{1p} - \theta) + 2 \sin \phi \sin \phi_{1p} \cos \phi \right] + \left[\frac{1}{j\omega r} \right] \sin \theta \sin \theta_{1p} \sin \phi \sin \phi_{1p} - \theta \right\}$	
	(140)
$H_\phi = \frac{-j\omega^2 \mu_0 \epsilon_0^2}{2\pi} \left(1 - \frac{z}{c}\right) \left\{ \left[\frac{2}{(j\omega r)^4} + \frac{2}{(j\omega r)^3} + \frac{1}{(j\omega r)^2} \right] \left[\sin \theta \sin \theta_{1p} (\cos \phi \cos \phi_{1p} - \theta) + 2 \sin \phi \sin \phi_{1p} \cos \phi \right] + \cos \theta \cos \theta_{1p} \cos \phi + \left[\frac{1}{(j\omega r)^3} + \frac{1}{(j\omega r)^2} \right] (\cos \theta \cos \theta_{1p} \cos \phi + \sin \theta \sin \theta_{1p} \cos \phi_{1p} \cos \phi_{1p}) + \left[\frac{1}{j\omega r} \right] \sin \theta \sin \theta_{1p} \sin \phi \sin \phi_{1p} - \theta \right\}$	
	(141)
$E_\theta = \frac{-j\omega^2 \mu_0 \epsilon_0^2}{2\pi} \left(1 - \frac{z}{c}\right) \left\{ \left[\frac{1}{(j\omega r)^3} + \frac{1}{(j\omega r)^2} \right] \sin \phi \left[3 \sin 2\theta \sin \theta_{1p} \cos \phi_{1p} - \theta \right] + 2(1 - 3 \sin^2 \theta) \cos \theta_{1p} + \left[\frac{1}{j\omega r} \right] \sin \phi \left[\sin 2\theta \sin \theta_{1p} \cos \phi_{1p} - \theta \right] - 2 \sin^2 \theta \cos \theta_{1p} \right\}$	
	(142)
$H_\theta = \frac{-j\omega^2 \mu_0 \epsilon_0^2}{2\pi} \left(1 - \frac{z}{c}\right) \left\{ \left[\frac{2}{(j\omega r)^4} + \frac{2}{(j\omega r)^3} + \frac{1}{(j\omega r)^2} \right] \sin \phi \left[3 \sin 2\theta \sin \theta_{1p} \cos \phi_{1p} - \theta \right] + 2(1 - 3 \sin^2 \theta) \cos \theta_{1p} + \left[\frac{1}{j\omega r} \right] \sin \phi \left[\sin 2\theta \sin \theta_{1p} \cos \phi_{1p} - \theta \right] - 2 \sin^2 \theta \cos \theta_{1p} \right\}$	

Vertical Polarization

Vertical Polarization

Horizontal Polarization

TABLE 9-2. FIELD EQUATIONS FOR THE COAXIAL SPACED LOOP

$$E_z = 0 \quad (41)$$

$$H_z = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] 2(1 - 3 \sin^2 \theta \sin^2 \phi) \quad (42)$$

$$\text{Vertical Polarisation} \begin{cases} E_\theta = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left[\frac{3}{(j\beta r)^3} + \frac{3}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \theta \sin 2\phi \\ H_\phi = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left[\frac{6}{(j\beta r)^4} + \frac{6}{(j\beta r)^3} + \frac{3}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin \theta \sin 2\phi \end{cases} \quad (43)$$

$$\text{Horizontal Polarisation} \begin{cases} E_\phi = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left[\frac{3}{(j\beta r)^3} + \frac{3}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin 2\theta \sin^2 \phi \\ H_\theta = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left[\frac{6}{(j\beta r)^4} + \frac{6}{(j\beta r)^3} + \frac{3}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin 2\theta \sin^2 \phi \end{cases} \quad (44)$$

TABLE 9-3. FIELD EQUATIONS FOR THE VERTICAL COPLANAR SPACED LOOP

$$E_z = \frac{+10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{2\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] \cos \theta \quad (45)$$

$$H_z = \frac{+10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] 3 \sin^2 \theta \sin 2\phi \quad (46)$$

$$\text{Vertical Polarisation} \begin{cases} E_\theta = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left\{ \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] 2(\cos 2\phi - \sin^2 \phi) - \left[\frac{2}{j\beta r} \right] \sin^2 \phi \right\} \sin \theta \\ H_\phi = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left\{ \left[\frac{6}{(j\beta r)^4} + \frac{6}{(j\beta r)^3} \right] \cos 2\phi + \left[\frac{2}{(j\beta r)^2} \right] (\cos 2\phi - \sin^2 \phi) - \left[\frac{2}{j\beta r} \right] \sin^2 \phi \right\} \sin \theta \end{cases} \quad (47)$$

$$\text{Horizontal Polarisation} \begin{cases} E_\phi = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left[\frac{3}{(j\beta r)^3} + \frac{3}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin 2\theta \sin 2\phi \\ H_\theta = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left[\frac{6}{(j\beta r)^4} + \frac{6}{(j\beta r)^3} + \frac{3}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \sin 2\theta \sin 2\phi \end{cases} \quad (48)$$

TABLE 9-4. FIELD EQUATIONS FOR THE HORIZONTAL COPLANAR SPACED LOOP

$$E_z = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] 2 \sin \theta \cos \phi \quad (49)$$

$$H_z = \frac{10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] 3 \sin 2\theta \sin \phi \quad (50)$$

$$\text{Vertical Polarisation} \begin{cases} E_\theta = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left[\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] 2 \cos \theta \cos \phi \\ H_\phi = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left[\frac{3}{(j\beta r)^4} + \frac{3}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right] 2 \cos \theta \cos \phi \end{cases} \quad (51)$$

$$\text{Horizontal Polarisation} \begin{cases} E_\phi = \frac{10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left\{ \left[\frac{2}{(j\beta r)^3} + \frac{2}{(j\beta r)^2} \right] (\cos 2\theta - \sin^2 \theta) - \left[\frac{1}{j\beta r} \right] \sin^2 \theta \right\} \sin \phi \\ H_\theta = \frac{-10^3 A r_0 g_0^{ju} \left(1 - \frac{z}{c}\right)}{4\pi} \left\{ \left[\frac{6}{(j\beta r)^4} + \frac{6}{(j\beta r)^3} \right] \cos 2\theta + \left[\frac{2}{(j\beta r)^2} \right] (\cos 2\theta - \sin^2 \theta) - \left[\frac{1}{j\beta r} \right] \sin^2 \theta \right\} \sin \phi \end{cases} \quad (52)$$

Some of the pattern variations which correspond to various conditions on the general field equations (38) and (39) for the general spaced loop are plotted in Figure 3. The patterns have been grouped to show pattern shape changes as a function of dipole orientation or type of spaced loop. Patterns at the extreme left therefore correspond to the vertical coplanar spaced loop and patterns at the extreme right correspond to the coaxial spaced loop. The patterns show variation with distance to the source in the vertical direction; those at the top correspond to the near field and those at the bottom to the far field. The patterns are also grouped according to polarization.

Starting with the coaxial spaced loop, (on the right in Fig. 3) as the dipoles are rotated so as to approach the configuration of the coplanar spaced loop two of the four lobes decrease in amplitude and two of the lobes increase. This effect occurs for both near and far field sources. However, for sources in the intermediate range (in the vicinity of one-third wavelength) the nulls are replaced by minima. These minima are in general not 90° apart.

It is evident that as the transition is made from the coaxial to the coplanar spaced loop four nulls are maintained if the source is in the near field, but the four nulls reduce to two if the source is in the far field. Polarities are not shown in Figure 3, however, the four lobes of the coaxial spaced loop pattern have alternating polarities while the two lobes of the coplanar spaced loop have the same polarity. This transition is evident in the far field patterns of Figure 3 for both vertical and horizontal polarization.

The patterns for horizontal polarization do not exhibit changes in shape due to distance to the source as in the case of vertical polarization. Therefore, the last row of patterns in Figure 3 applies to any source distance which is at least greater than the spacing between loops.

The similarity between the two patterns corresponding to $\phi_{1p} = 90^\circ$, $\phi_{1p} = 67\frac{1}{2}^\circ$ for vertical polarization indicates that the modification to the pure coaxial spaced loop pattern, which is produced by a slight change in the angle ϕ_{1p} , is very similar to the distortion due to an omnidirectional component. This is interesting in that in the past similarly appearing distorted spaced loop patterns have been experimentally obtained which did not seem to be related to vertical pickup in the antenna system.

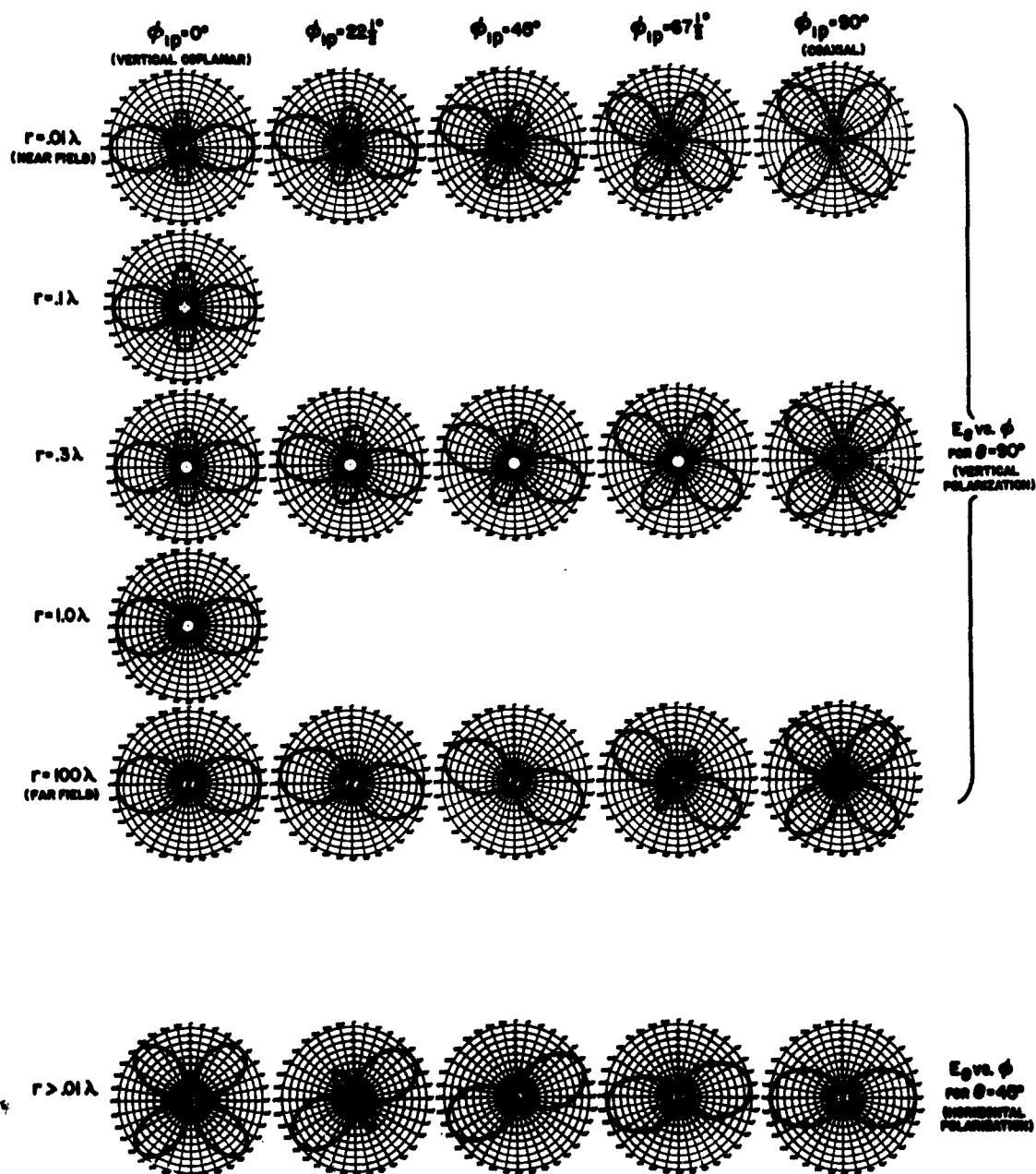


FIGURE 3.
 AZIMUTH PLANE PATTERNS FOR THE SPACED LOOP.
 $\theta_{ip} = 90^\circ$ (LOOPS VERTICAL)

10. Combinations of Spaced Loop and Simple Loop Antennas

An arrangement of three loops in certain cases produces patterns which are symmetrical about only one axis and hence may be used as D/F antennas without sense ambiguity. The three-loop antenna developed under contract NObsr-64585 is an example and corresponds to combining the equations from Table 9-2 with those from Table 9-6. Details of the phasing and amplitude difference problems which arise in this case were reported in the final report of the same contract¹¹.

For vertical polarization the total response will be the sum of the E_θ components; for horizontal polarization it will be the sum of the E_ϕ components. Considering only the response to vertical polarization, if the spaced loop and simple loop outputs are simply added, the voltage presented to the receiver will be the sum of equations (42), (see Table 9-2), and (88), (see Table 9-6). This corresponds to the three-loop configuration where the simple loop axis is perpendicular to the coaxial spaced loop axis.

For this case there are two inverse distance terms in the simple loop response equation and three inverse distance terms in the spaced loop response equation. This difference produces both an amplitude and a phase change in the total response as a target source is moved from the far to the near field. Since all target sources are located in the far field (that is, they are at least many wavelengths away) while local reradiators such as are found on board ship are generally in the near field, it is evident that the complete response must be considered.

Examination of the equations in Table 9-1 shows that the near field E or H terms are all in phase, that is, the electric field of the simple loop is in phase with the electric field of the spaced loop when the source is near by. This means that when the three-loop antenna is tested with a nearby source, such as the transmission line method in a screen room, the outputs may be simply added with zero phase shift. On the other hand the equations show that when the source is in the far field, the response of the simple loop is in quadrature with the response of the spaced loop and a 90-degree phase shift is required to combine outputs to produce a proper D/F pattern. For sources at intermediate distances, the phase difference varies between 0 and 90 degrees. It is effectively 90 degrees for all distances greater than three wavelengths and effectively zero for all distances less than 6×10^{-2} wavelengths. It is approximately 45 degrees at one-third of a wavelength.

11. Travers, Douglas N., et al., "Methods for the Reduction of Reradiation Errors in Naval High Frequency Shipboard Direction Finding," Final Development Report, Southwest Research Institute, 1 January 1961.

If the spaced loop is used alone without a simple loop, equations (42) and (67) (vertical polarization, coaxial spaced loop) show that the shape is independent of the distance to the source. Experimental evidence has shown that even when the sources are very close such that the distance is equal to the spacing between loops the shape of the pattern is modified only slightly. The general shape in this case remains the same, that is, four nulls are obtained 90 degrees apart.

Another point evident from the equations is that the near field terms are independent of frequency. This means that if testing is conducted in a screen room, the variation in gain required to mix the signals with equal amplitude (or to provide some degree of sense) will appear to be less than that which is actually required when the target is distant. This effect will be more noticeable if the operating frequency range is increased.

It is also evident that the simple loop far field is proportional to frequency while the spaced loop far field is proportional to frequency squared. The relationship is such that for similar sized small antennas the far field of the spaced loop is much less than that of the simple loop at low frequencies. (This point is considered in more detail in sections 15 and 17 on signal-to-noise ratio). This requires that the isolation between circuits of the two antennas be relatively high prior to combining.

Another particularly important point is that the geometry of the coaxial spaced loop will not be critical for a source in the near field but will be so for a source in the far field. The reason for this is explained in the final report of contract NObsr-64585, in terms of the field equations, however, it is related to the fact that the output for the far field source results from a field phase difference across the array whereas for nearby sources the output results from amplitude change across the array. It is thus necessary when constructing spaced loops where construction tolerances are in doubt, to evaluate pattern quality with a far field source. It is quite possible to experimentally obtain near field spaced loop patterns which are apparently perfect, from an antenna which when tested in the far field, produces so much distortion that only a dipole mode can be observed.

Another combination of three loops which will produce a pattern with one axis of symmetry is obtained by rotating the simple loop 90 degrees from the position considered in the last example. This produces three parallel coaxial loops. The response may be studied by combining equations (42) and (91) for the electric field and (67) and (104) for the magnetic field. There is little theoretical difference between this case and the one previously considered.

The third possible combination with the coaxial spaced loop where the simple loop axis is vertical is not important for cases where it is desired to receive only vertical polarization. The horizontal loop does not respond to vertical polarization.

There are a number of combinations possible with the coplanar spaced loop. One of the most important of these is the combination of the vertical coplanar spaced loop and the simple loop aligned with its axis along the X-axis. This arrangement results in three parallel loops all lying in the same plane. This antenna was first investigated by Friis prior to 1925¹².

The response of this combination may be investigated for vertical polarization by combining equations (45) and (88) for the electric field and equations (70) and (101) for the magnetic field. Most of the comments made previously concerning the coaxial spaced loop are also appropriate for this combination with the exception of pattern shape as a function of distance to the source. It is evident from equations (45) and (70) that the shape of the pattern undergoes a change as the source is moved from the far field to the near field. This is in addition to the amplitude and phase changes. Furthermore, the change is different for the E and H fields. The far field response is proportional to $\sin^2\phi$ as was shown by Friis so that the pattern has two nulls and two maximums, both maximums being of the same phase. In the near field the electric component is proportional to $(\cos 2\phi - \sin^2\phi)$ and the magnetic component is proportional to $\cos 2\phi$, thus the response pattern has more than two nulls for sources in the near field.

An antenna of this type will not respond to local shipboard reradiators in the same manner that it responds to distant targets. It is also true that if the antenna is phased with a simple loop, as was done by Friis, the response of the spaced loop will change according to equations (45) or (70) but the response of the simple loop will change according to equations (88) and (101). This occurs in such a way as to produce no change in the shape of the loop pattern, but a change in the spaced loop will occur. Thus, the basic assumption made by Friis that this combination can be used to produce a null in the back direction of the forward lobe is only valid for far field sources. For instance, if such a combination is designed to produce a null

12. Friis, H. T., "A New Directional Receiving System," Proc. IRE, Vol. 13, December 1925, pp. 685-707.

in the backward direction on the basis of a far field analysis in order to reject reradiation from a local source such as a reradiating mast, the null will not be produced and the antenna will respond to the mast.

Other combinations of three loops may be arranged for either null or max type direction finders for either vertical or horizontal polarization. In these instances it is possible to arrange the loops for response to only either one polarization or the other but it does not appear to be possible to make an arrangement which responds equally to both polarizations. It is possible, however, to have the antenna respond with a null to both vertical and horizontal polarization if only the spaced loop mode is used.

For instance the coaxial spaced loop has four nulls for vertical polarization and two nulls for horizontal polarization. These combine in such a manner that it has two nulls which remain fixed for any polarization. Similarly in the far field the vertical coplanar spaced loop has two nulls for vertical polarization and four nulls for horizontal polarization such that two nulls are always maintained for any polarization. When used in combination with simple loops, however, these advantages are lost because the simple loop has two nulls for vertical polarization and two nulls for horizontal polarization; however, they do not coincide [see equations (88) and (89)].

11. Effective Height of the Spaced Loop

The effective height or effective length¹³ of an antenna is a quantity used to indicate the effectiveness of the antenna as a radiator or as a collector of electromagnetic energy in a certain direction. The term is often used in connection with electrically small receiving antennas such as loops and as such is often defined as the constant of proportionality between induced voltage V and incident field intensity E

$$V = h_e E \quad (114)$$

The effective height may also be derived from consideration of the antenna as a transmitting source. In this case the effective height is the length of an equivalent linear antenna with uniform current distribution which radiates the same field as the antenna in question, in the direction perpendicular to its length. Both transmitting and receiving effective heights are equal, and the quantity is independent of antenna losses or coupling networks to the receiver.

13. The term effective height is meant in the original sense as in equation (114) and is not to be confused with the height of the antenna above ground.

The radiated far region field of a current element of length dl in a direction perpendicular to its length is given by the far field term of equation (110)¹⁴. Neglecting phase this field is

$$E = \frac{IL\beta^3}{4\pi\epsilon\omega\beta r} = \frac{IL\omega^2}{4\pi\epsilon\omega c^2 r} = \frac{I\omega dl}{4\pi\epsilon c^2 r} \quad (115)$$

Therefore, the field intensity produced by an antenna having an effective height h_e will be

$$E = \frac{Ih_e\omega}{4\pi\epsilon c^2 r} \quad (116)$$

where I is now the current at the terminals of the antenna and also the uniform current in the element.

Equation (116) may be solved for the effective height

$$h_e = \frac{4\pi\epsilon c^2 r E}{I\omega} \quad (117)$$

This is equivalent to

$$h_e = \frac{4\pi r E}{I\omega\mu_0} \quad (118)$$

From the field equations derived in the previous sections, the effective heights may now be derived by substitution into equation (118). For instance, for vertical polarization (E_θ), substituting the far field term from equation (38) yields the effective height of the general spaced loop

$$h_e = - \frac{4\pi r}{I\omega\mu_0} \cdot \frac{\beta^3 A r d \omega \mu}{j 2\pi \beta r} \sin\theta \sin\phi \sin\theta_{1p} \sin(\phi_{1p} - \phi) \quad (119)$$

and in free space

$$|h_e| = \left| 2\beta^2 A r d \sin\theta \sin\phi \sin\theta_{1p} (\sin\phi_{1p} \cos\phi - \cos\phi_{1p} \sin\phi) \right| \quad (120)$$

For the coaxial spaced loop $\theta_{1p} = \phi_{1p} = 90^\circ$ and the effective height is

$$|h_e| = \left| 2\beta^2 A r d \sin\theta \sin\phi \cos\phi \right| = \left| \beta^2 A r d \sin\theta \sin 2\phi \right| \quad (121)$$

14. Jordan, E. C., Electromagnetic Waves and Radiating Systems, New York, Prentice-Hall, Inc., 1950, p. 305.

This is a maximum in the direction $\theta = 90^\circ$, $\phi = 45^\circ$ so that in this direction equation (121) becomes

$$h_e = \beta^2 A r_d \quad (122a)$$

or for an antenna with a core having a permeability μ_r

$$h_e = \beta^2 A r_d \mu_r \quad (122b)$$

This is the effective height for a series connected coaxial spaced loop and is a well-known formula^{15, 16}. The series connected case results in this analysis from the form of equation (25) where potentials were added. This is equivalent to adding (or subtracting) the voltages induced in the two loops. The parallel connected effective height is therefore exactly one-half the series connected value. Virtually all practical coaxial spaced loops are differentially connected in parallel, therefore, they have the effective height

$$h_e = \frac{2\pi^2 A r_d \mu_r}{\lambda^2} \quad (123)$$

For a multiturn coaxial spaced loop of N turns on each loop, with the turns connected in series and the two loops connected in parallel, the effective height becomes

$$h_e = \frac{2\pi^2 A r_d \mu_r N}{\lambda^2} \quad (124)$$

The quantity $2A r_d$ is sometimes referred to as V, the volume of the spaced loop, so that equation (124) may also be expressed as

$$h_e = \frac{\pi^2 V N \mu_r}{\lambda^2} \quad (125)$$

Similarly, the effective height of a parallel opposition connected vertical coplanar spaced loop is

$$h_e = \frac{2\pi^2 V N \mu_r}{\lambda^2} \quad (126)$$

15. Bond, Donald S., Radio Direction Finders, New York, McGraw-Hill Book Co., Inc. 1944.

16. Anonymous, "Radio Direction Finding," War Department TM 11-476, Washington, July 1947.

The effective height of a horizontal coplanar spaced loop is zero for vertical polarization; however, for the E_ϕ component (representing horizontal polarization) the effective height is the same as the vertical coplanar spaced loop for vertical polarization, or equal to equation (126). The factor μ_r in the above equations may be taken as the effective permeability of a core (for instance ferrite) inserted through the loops of the antenna ($\mu_{air} = 1$).

12. Radiation Resistance of the Spaced Loop

The importance of the radiation resistance lies in the fact that it may be used to determine gain and the signal-to-noise ratio if the effective height is known. It is also instructive in illustrating the difficulty of impedance matching the small spaced loop antenna for maximum power transfer.

The radiation resistance of the spaced loop antenna may be easily found by the Poynting Vector Method of integration over a large sphere. The radiation resistance is given by the ratio of the power flowing through the sphere to the square of the RMS terminal current of the antenna. Thus

$$R_r = \frac{W}{I_{rms}^2} = \frac{Z_0 \iint H^2 ds}{2I_{rms}^2} \quad (127)$$

where ds is the area increment on the surface of the sphere, and H is the complete magnetic field of the antenna in the far region. Since the radial magnetic field is zero in the far region, H is given by

$$H = \vec{H}_\theta + \vec{H}_\phi = \hat{a}_\theta |\vec{H}_\theta| + \hat{a}_\phi |\vec{H}_\phi| \quad (128)$$

Therefore

$$H = \sqrt{|H_\theta|^2 + |H_\phi|^2} \quad (129)$$

or

$$H^2 = |H_\theta|^2 + |H_\phi|^2 \quad (130)$$

Substituting equation (130) into equation (127) yields for free space

$$R_r = \frac{\sqrt{\frac{\mu_0}{\epsilon_0}} \int_0^{2\pi} \int_0^\pi (|H_\theta|^2 + |H_\phi|^2) ds}{2I_{rms}^2} \quad (131)$$

The far field terms, H_θ and H_ϕ , are obtained from equations (63) and (64)

$$H_\theta = \frac{-\beta^4 A l r_d e^{j\omega(t - \frac{r}{c})}}{2\pi j \beta r} [\cos \theta \sin \theta_{1p} \cos(\phi_{1p} - \phi) - \sin \theta \cos \theta_{1p}] \sin \phi \sin \theta \quad (132)$$

$$H_\phi = \frac{-\beta^4 A l r_d e^{j\omega(t - \frac{r}{c})}}{2\pi j \beta r} [\sin \theta \sin \theta_{1p} \sin \phi \sin(\phi_{1p} - \phi)] \quad (133)$$

Squaring and adding yields

$$\begin{aligned} |H_\theta|^2 + |H_\phi|^2 &= \frac{\beta^8 l^2 A^2 r_d^2}{4\pi^2 (\beta r)^2} [\sin^2 \phi \sin^2 \theta (\cos^2 \theta \sin^2 \theta_{1p} \cos^2(\phi_{1p} - \phi) \\ &\quad + \sin^2 \theta \cos^2 \theta_{1p} - 2 \sin \theta \cos \theta \sin \theta_{1p} \cos \theta_{1p} \cos(\phi_{1p} - \phi) \\ &\quad + \sin^2 \theta_{1p} \sin^2(\phi_{1p} - \phi))] \quad (134) \end{aligned}$$

The area increment is

$$ds = r^2 \sin \theta \, d\theta \, d\phi \quad (135)$$

Therefore, equation (131) becomes

$$\begin{aligned} R_r &= \frac{1}{2} \sqrt{\frac{\mu_0}{\epsilon_0}} \left[\frac{\beta^3 A r_d l}{2\pi I_{rms}} \right]^2 \int_0^{2\pi} \int_0^\pi \sin^2 \phi \sin^3 \theta [\cos^2 \theta \sin^2 \theta_{1p} \cos^2(\phi_{1p} - \phi) \\ &\quad + \sin^2 \theta \cos^2 \theta_{1p} - \sin 2\theta \sin \theta_{1p} \cos \theta_{1p} \cos(\phi_{1p} - \phi) \\ &\quad + \sin^2 \theta_{1p} \sin^2(\phi_{1p} - \phi)] \, d\theta \, d\phi \quad (136) \end{aligned}$$

The integration is straightforward and results in the term $8\pi(2 - \sin^2 \theta_{1p} \sin^2 \phi_{1p})/15$. Therefore, the radiation resistance of the series connected general spaced loop is

$$R_r = \sqrt{\frac{\mu_0}{\epsilon_0}} \left[\frac{\beta^3 A r_d l}{2\pi I_{rms}} \right]^2 \frac{4\pi}{15} (2 - \sin^2 \theta_{1p} \sin^2 \phi_{1p}) \text{ ohms} \quad (137)$$

This is equivalent to

$$R_r = 9.85 \times 10^5 \left(\frac{A_{rd}}{\lambda^3} \right)^2 (2 - \sin^2 \theta_{1p} \sin^2 \phi_{1p}) \text{ ohms} \quad (138)$$

For parallel connected loops the radiation resistance will be one-fourth the series connected value. For the coaxial spaced loop, $\theta_{1p} = 90^\circ$, $\phi_{1p} = 90^\circ$, and the radiation resistance becomes

$$R_r = 9.85 \times 10^5 \left(\frac{A_{rd}}{\lambda^3} \right)^2 \text{ ohms (series connected)} \quad (139)$$

$$R_r = 2.46 \times 10^5 \left(\frac{A_{rd}}{\lambda^3} \right)^2 \text{ ohms (parallel connected)} \quad (140)$$

For the coplanar spaced loop (either vertical or horizontal) the radiation resistance is

$$R_r = 1.969 \times 10^6 \left(\frac{A_{rd}}{\lambda^3} \right)^2 \text{ ohms (series connected)} \quad (141)$$

$$R_r = 4.92 \times 10^5 \left(\frac{A_{rd}}{\lambda^3} \right)^2 \text{ ohms (parallel connected)} \quad (142)$$

These values are exactly twice the values for the coaxial spaced loop.

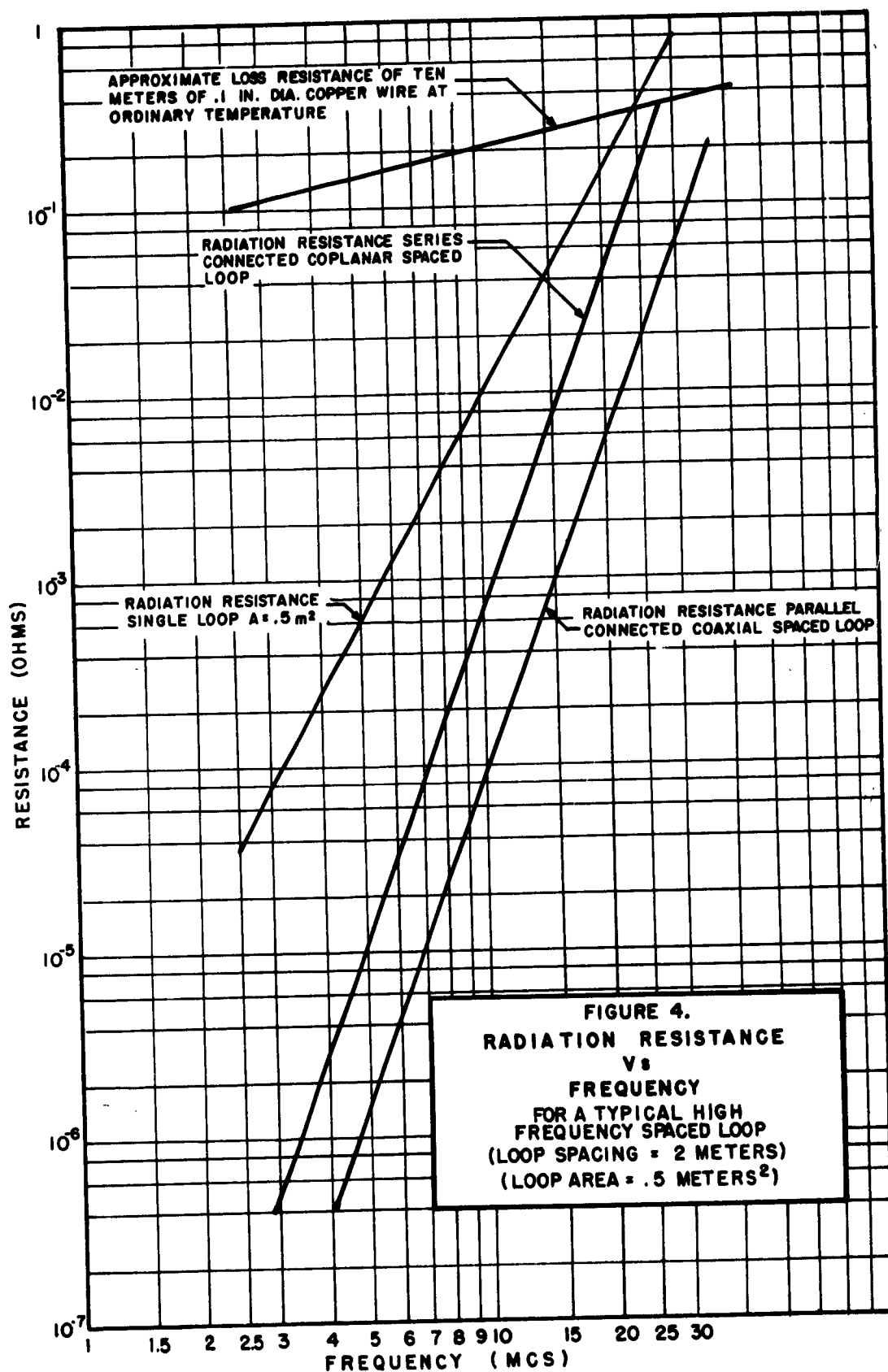
The radiation resistance for the spaced loop varies as the inverse sixth power of the wavelength. Thus even though the constant coefficient appears to be large, the radiation resistance is quite small for $A_{rd} \ll \lambda^3$. In Figure 4 the radiation resistance of two spaced loops of typical size is plotted versus frequency. It is evident that impedance matching of the radiation resistance to the receiver input is not possible by any ordinary means. It is further evident that the radiation resistance is negligible with respect to loss resistance for ordinary conditions, and typical wire sizes.

13. Gain of the Spaced Loop

The gain of an antenna is related to the effective height and radiation resistance by the equation¹⁷

$$g = \frac{120\pi^2}{R_r} \left(\frac{h_e}{\lambda} \right)^2 \quad (143)$$

17. Jordan, Edward C., Electromagnetic Waves and Radiating Systems, New York, Prentice-Hall, Inc., 1950, p. 417.



Substituting equations (120) and (137) yields the gain of the general spaced loop for vertical polarization

$$g = \frac{15}{8} \cdot \frac{\sin^2 \theta \sin^2 \theta_{1p} (\sin^2 \phi_{1p} \sin^2 2\phi - 2 \sin 2\phi_{1p} \sin 2\phi \sin^2 \phi + 4 \cos^2 \phi_{1p} \sin^4 \phi)}{(2 - \sin^2 \theta_{1p} \sin^2 \phi_{1p})} \quad (144)$$

For the coaxial spaced loop $\theta_{1p} = 90^\circ$, $\phi_{1p} = 90^\circ$ and equation (144) becomes

$$g = \frac{15}{8} \sin^2 \theta \sin^2 2\phi \quad (145)$$

This is maximum at $\theta = 90^\circ$, $\phi = 45^\circ$ hence in this direction the gain is

$$g = \frac{15}{8} \quad (146)$$

For the coplanar spaced loop $\theta_{1p} = 90^\circ$, $\phi_{1p} = 0$ hence the gain for vertical polarization is

$$g = \frac{30}{8} \sin^2 \theta \sin^4 \phi \quad (147)$$

This is maximum at $\theta = 90^\circ$, $\phi = 90^\circ$ hence in this direction the gain is¹⁸

$$g = \frac{15}{4} \quad (148)$$

The above values apply whether the loops are connected in series or parallel.

14. Effective Area of the Spaced Loop

The effective area is another parameter for expressing the effectiveness of a receiving antenna. It is defined in terms of the gain of an antenna by the relation¹⁹

$$A_e = \frac{\lambda^2 g}{4\pi} \quad (149)$$

Using this relation it can be shown that the effective area is the ratio of power available in the antenna to the power per unit area of the appropriately

18. In agreement with Shelkunoff, Antennas Theory and Practice, page 199, No. 6-1.6.

19. Jordan, Edward C., Electromagnetic Waves and Radiating Systems, N. Y., Prentice-Hall, Inc., 1950, p. 416. Also Shelkunoff, S., Antennas Theory and Practice, N. Y., John Wiley, p. 185.

polarized incident wave. That is, the received power is equal to the power flow through an area equal to the effective area of the antenna. Substituting equation (143) into equation (149) yields

$$A_e = \frac{30\pi h_e^2}{R_r} \quad (150)$$

or for vertical polarization

$$A_e = \frac{15\lambda^2}{32\pi} \quad \begin{array}{l} \text{square meters for the lossless} \\ \text{coaxial spaced loop} \end{array} \quad (151)$$

$$A_e = \frac{15\lambda^2}{16\pi} \quad \begin{array}{l} \text{square meters for the lossless} \\ \text{coplanar spaced loop} \end{array} \quad (152)$$

The above relations hold for either series or parallel connected loops. The power received is therefore

$$P_r = P A_e \quad (153)$$

where P = power in an incident wave E in watts per square meter, or

$$P = \frac{E^2}{Z_0} = \frac{E^2}{120\pi} \text{ watts/m}^2 \quad (154)$$

The power available in the antenna is therefore

$$P_r = \frac{E^2 A_e}{120\pi} = \frac{(E h_e)^2}{4 R_r} = \left(\frac{E h_e}{2} \right)^2 \frac{1}{R_r} \text{ watts} \quad (155)$$

The power which is ordinarily delivered to the receiver is

$$P_{in} = \left(\frac{E h_e}{2} \right)^2 \frac{1}{R_{in}} \text{ watts} \quad (156)$$

where R_{in} is the equivalent receiver input resistance. Hence the efficiency of the power transfer to the receiver is

$$\text{eff.} = \frac{R_r}{R_{in}} \times 100\% \quad (157)$$

From examination of Figure 4, it is evident that this efficiency must be in the range of much less than 1 percent for most frequencies. This is simply another way of showing that, practically, the spaced loop is not an efficient antenna. However, for receiving applications, it is more important to consider the signal-to-noise ratio than maximum power transfer. This is covered in the next section.

15. Signal-to-Noise Ratio of the Spaced Loop Antenna

It is of interest to know the maximum signal-to-noise ratio obtainable with the spaced loop and to determine the weakest field strength which can be received in the ideal case and in the practical case. The material which follows is based on equations derived by Burgess²⁰.

In computing the signal-to-noise ratio in receiving systems, it is necessary to estimate the noise arising in the antenna. Thermal noise is given by Nyquist's theorem²¹ which states that the mean square fluctuation e. m. f. in the frequency band ν to $\nu + d\nu$ appearing in an impedance $R + jX = Z$ is given by

$$\overline{de} = \sqrt{4kTRd\nu} \quad (158)$$

where k = Boltzmann's constant = 1.374×10^{-23} joules per degree Kelvin, and T is the absolute temperature in degrees Kelvin. The value R is assumed to be constant over the bandwidth $d\nu$; if R is a function of frequency, a more general form of equation (158) is required²². However, for the present analysis we will be concerned with bandwidths which are less than one-half of one percent of the center frequency, so that while the previously calculated radiation resistances are rapidly changing with frequency, the changes are small over the bandwidths considered and equation (158) is closely approximated.

Burgess shows that the radiation resistance of an antenna can apparently be the source of thermal noise and that it obeys Nyquist's theorem when the aerial is in radiative equilibrium with its surroundings²³. Thus

$$\overline{de_r} = \sqrt{4kTR_r d\nu} \quad (159)$$

-
20. Burgess, R. E., "Noise in Receiving Aerial Systems," Proceedings of Physical Society, Vol. 53, May 1941, p. 293.
 21. Nyquist, H., "Thermal Agitation of Electric Charge in Conductors," Physical Review, Vol. 32, July 1928, p. 110.
 22. For instance see Terman, F., "Radio Engineers Handbook," p. 476.
 23. Burgess points out that this statement is in disagreement with other authors in papers published prior to his 1941 paper. The validity of Burgess' paper now seems well established however.

The equivalent noise temperature T_r of the radiation resistance R_r is defined as that temperature which must be ascribed to R_r so as to give the value of the received noise electromotive force when substituted in Nyquist's equation, so that

$$\overline{de_r} = \sqrt{4kT_r R_r dv} \quad (160)$$

Thus an antenna in an enclosure at a uniform temperature, T , is in effect the source of a thermal e. m. f. whose value corresponds to the condition $T_r = T$. If the antenna temperature T_a is not equal to T of the enclosure, radiative equilibrium does not exist and there will be an unbalanced energy flow between the antenna and its surroundings in the sense which tends to equalize T_a and T . It is concluded that the radiation resistance of an antenna in free space has an equivalent noise temperature of zero; expressed otherwise there is no other source of radiation present to return energy to the antenna and thus to induce a fluctuation e. m. f.

T_r will in general be independent of the bandwidth, but it will not be independent of antenna gain or effective height if the noise source is not uniformly distributed in all directions. Burgess points out therefore that T_r will, in general, exceed T , and, in fact, high frequency measurements by Jansky have shown that T_r is always greater than T and can exceed 1000 T in unfavorable circumstances.

Assuming that the proper T_r is attributed to the radiation resistance, then the noise e. m. f. of the lossless antenna is

$$e_r = \sqrt{4kT_r R_r B} \quad (161)$$

where B is the bandwidth. Now the voltage output of an antenna is related to an incident signal field E by

$$V = Eh_e \quad (162)$$

Hence for unity signal-to-noise ratio at the output terminals of a lossless antenna the incident signal field strength must be

$$E = \frac{\sqrt{4kT_r R_r B}}{h_e} \quad (163)$$

For the general spaced loop antenna the ratio of $\sqrt{R_r}$ to h_e is given by equations (137) and (120), thus:

$$\frac{\sqrt{R_r}}{h_e} = \frac{4 \sqrt{\frac{\mu_0}{\epsilon_0}} \left[\frac{\beta^3 A_r d I}{2\pi I_{rms}} \right] \frac{2\sqrt{\pi}}{\sqrt{30}} \cdot \sqrt{2 - \sin^2 \theta_{lp} \sin^2 \phi_{lp}}}{2\beta^2 A_r d \sin \theta \sin \phi \sin \theta_{lp} (\sin \phi_{lp} \cos \phi - \cos \phi_{lp} \sin \phi)} \quad (164)$$

Note that the size of the spaced loop cancels out so that equation (164) simplifies to

$$\frac{\sqrt{R_r}}{h_e} = \frac{4\pi \sqrt{(2 - \sin^2 \theta_{lp} \sin^2 \phi_{lp})}}{\lambda \sin \theta \sin \phi \sin \theta_{lp} \sin(\phi_{lp} - \phi)} \quad \begin{array}{l} \text{series or} \\ \text{parallel} \end{array} \quad (165)$$

Thus the weakest field which a lossless spaced loop antenna can receive is

$$E_{(o)min} = \frac{4\pi}{\lambda} \frac{\sqrt{4kT_r B (2 - \sin^2 \theta_{lp} \sin^2 \phi_{lp})}}{\sin \theta \sin \phi_{lp} \sin \theta \sin(\phi_{lp} - \phi)} \quad (166)$$

It is interesting to note that this field is independent of the size ($A_r d$) of the spaced loop, therefore, it is apparent that the designers' desire to make the spaced loop large is to overcome effects due to noise in the associated circuits and the antenna loss resistance.

For the coaxial spaced loop ($\theta_{lp} = \phi_{lp} = 90^\circ$) equation (166) becomes

$$E_{(o)min} = \frac{8\pi}{\lambda} \frac{\sqrt{4kT_r B}}{\sin \theta \sin 2\phi} \quad (167)$$

The effective height is maximized in the directions $\theta = 90^\circ$, $\phi = 45^\circ$ hence

$$E_{(o)min} = \frac{16\pi \sqrt{kT_r B}}{\lambda} = 8\beta \sqrt{kT_r B} \quad (168)$$

The corresponding expression for the vertical coplanar spaced loop is

$$E_{(o)min} = 8\beta \sqrt{2kT_r B} \quad (169)$$

For typical conditions of current interest assume

$$\begin{aligned} \lambda &= 100 \text{ meters (} f = 3 \text{ mc)} \\ T_r &= 290^\circ \text{ Kelvin} \\ B &= 10 \times 10^3 \text{ cycles} \end{aligned}$$

Then the theoretical minimum fields at 290°K (16.8°C) for either series or parallel connected spaced loops are

$$E_{(o)\min} = .00317 \mu\text{v/meter (coaxial)} \quad (170)$$

$$E_{(o)\min} = .00217 \mu\text{v/meter (coplanar)} \quad (171)$$

Field strengths of these values will probably never be observed in practice for at least three reasons, (1) the antenna will contain some loss resistance which will be a source of noise and, in general, the value of R_r for a spaced loop will be small compared to this loss, (2) the coupling circuits and the first amplifier will be sources of noise which, in general, will be large compared to R_r noise, and (3) the value of T_r will often be above the assumed value of $T = 290^\circ\text{K}$ because of noise sources in the environment of the antenna. It is the purpose of the remainder of this section to estimate the degree to which these factors affect spaced loop signal-to-noise ratio in practical cases.

Consider the general case of an antenna (in this case a spaced loop) coupled to an amplifier as shown in Figure 5.

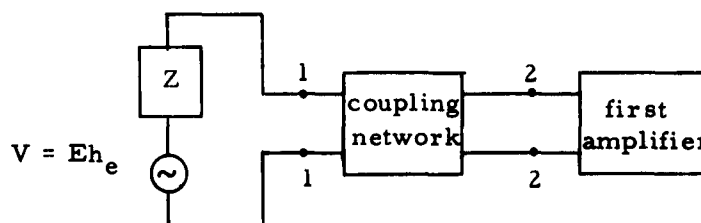


FIGURE 5

The coupling network is linear and passive. It may contain resistances, tuned circuits, etc., including loss resistances. The first amplifier may be any active network with sufficient gain and with an equivalent input impedance of $R_{in} + jX_{in}$ where R_{in} is a noise source.

The mean square noise e. m. f. at terminals 2-2 is given by

$$\overline{e_n^2} = 4kB \left[n_a T_r R_r + T \sum_s n_s^2 R_s \right] \quad (172)$$

where R_s is the resistive component of the sth impedance, element Z_s of the network including loss resistance in the antenna but excluding radiation

resistance R_r of the antenna, n_s = voltage transfer ratio from the impedance Z_s to terminals 2-2, n_a = voltage transfer ratio from the antenna to terminals 2-2, and T = temperature of the resistances in the coupling network.

Amplifier noise may be taken into account by adding a term $4kTBR_v$ when R_v is the equivalent noise resistance of the amplifier input, thus equation (172) becomes

$$\bar{e}_n = 2 \sqrt{kB} [n_a^2 T_r R_r + \sum_s n_s^2 R_s + TR_v]^{1/2} \quad (173)$$

The signal-to-noise ratio is therefore the ratio of equation (162) to equation (173), hence

$$\rho = \frac{\text{received signal}}{\text{received noise}} = \frac{V}{\bar{e}_n} = \frac{E_h e n_a}{2 \sqrt{kB} [n_a^2 T_r R_r + T \sum_s n_s^2 R_s + TR_v]^{1/2}} \quad (174)$$

When the antenna is lossless, the coupling network is lossless and the first amplifier is noise free, the absolute maximum value of the signal-to-noise ratio is obtained. This value is ρ_0 where

$$\rho_0 = \frac{E_h e}{2 \sqrt{kBT_r R_r}} \quad (175)$$

This signal-to-noise ratio is one to one at the field strengths mentioned previously. Substituting equation (175) into equation (174) yields the practical signal-to-noise ratio

$$\rho = \frac{\rho_0}{\left[1 + \frac{T(\sum_s n_s^2 R_s + R_v)}{T_r R_r n_a^2} \right]^{1/2}} \quad (176)$$

Substituting equation (175) into equation (176) and substituting the spaced loop values for h_e and R_r , equations (120) and (139), yields the signal-to-noise ratio of the one-turn general spaced loop in free space

$$\rho = \frac{n_a E \beta^2 A r_d \sin \theta \sin \phi \sin \theta_{1p} \sin(\phi_{1p} - \phi) \lambda^3}{\sqrt{kB} [T_r 9.85 \times 10^5 (A r_d)^2 (2 - \sin^2 \theta_{1p} \sin^2 \phi_{1p}) n_a^2 + T(\sum_s n_s^2 R_s + R_v) \lambda^6]} \quad (177a)$$

Note this is stated for series connected loops. When the loops are parallel connected the signal-to-noise ratio will be:

$$\rho = \frac{n_a E \beta^2 A r_d \sin \theta \sin \phi \sin \theta_{1p} \sin(\phi_{1p} - \phi) \lambda^3}{2 \sqrt{kB} [T_r 2.46 \times 10^5 (A r_d)^2 (2 - \sin^2 \theta_{1p} \sin^2 \phi_{1p}) n_a^2 + T(\Sigma n_s^2 R_s + R_v) \lambda^6]} \quad (177b)$$

16. Minimum Observable Field Strengths

Under practical conditions, for a unity signal-to-noise ratio ($\rho = 1$), the minimum field which can be observed is found by solving equation (174), thus

$$E_{\min} = \frac{2 \sqrt{kB T_r R_r} \left[1 + \frac{T}{T_r} \left(\frac{\Sigma n_s^2 R_s + R_v}{R_r n_a^2} \right) \right]^{1/2}}{h_e} \quad (178)$$

or

$$E_{\min} = \left[1 + \frac{T}{T_r} \left(\frac{\Sigma n_s^2 R_s + R_v}{R_r n_a^2} \right) \right]^{1/2} \cdot E_{(o)\min} \quad (179)$$

The loss resistances can be accounted for by substitution of appropriate values into R_s while the amplifier noise can be accounted for by R_v .

When the radiation resistance is given by equation (138) and $E_{(o)\min}$ by equation (166), equation (179) becomes

$$E_{\min} = \frac{4\pi}{\lambda} \frac{\sqrt{2kB}}{\sin \theta \sin \phi \sin \theta_{1p} \sin(\phi_{1p} - \phi)} \left[T_r (2 - \sin^2 \theta_{1p} \sin^2 \phi_{1p}) + \frac{T(\Sigma n_s^2 R_s + R_v) \lambda^6}{n_a^2 9.85 \times 10^5 (A r_d)^2} \right]^{1/2} \quad (180)$$

Note that equation (180) is for the general series connected spaced loop except that the polarization is vertical. For a parallel connected coaxial spaced loop the above becomes

$$E_{\min} = \frac{16\pi \sqrt{kB}}{\lambda \sin \theta \sin 2\phi} \left[T_r + \frac{T(\Sigma n_s^2 R_s + R_v) \lambda^6}{n_a^2 2.46 \times 10^5 (A r_d)^2} \right]^{1/2} \frac{\text{volts}}{\text{meter}} \quad (181)$$

a. An Aperiodic Spaced Loop with a Transistor Amplifier

Consider a practical circuit consisting of a coaxial spaced loop formed by four loops connected as one channel of an 8-loop array (two spaced loops at right angles and connected in parallel sum) and connected directly into a transistor amplifier. Let the transistor amplifier have a gain g and an equivalent noise input resistance R_V which includes the input resistance R_{in} of the amplifier circuit. Then in the direction of maximum reception equation (181) becomes

$$E_{min} = \frac{16\pi\sqrt{kB}}{\lambda} \left[T_r + \frac{(g^2 T_s R_s + T_V R_V) \lambda^6}{1.23 \times 10^5 g^2 (A_{rd})^2} \right]^{1/2} \frac{\text{volts}}{\text{meter}} \quad (182)$$

The spaced loop radiation resistance is one-half that of the single pair of loops, hence the factor 1.23×10^5 in the denominator. The 8-loop array now under experimental investigation has a diameter of 17 feet and loops with an area of 4 square feet each. Assuming $T_s = T_V = 290^\circ$, $k = 1.374 \times 10^{-23}$, and a frequency of 3 megacycles, equation (182) becomes

$$E_{min} = .335 \sqrt{kB} \left[T_r + \frac{(g^2 R_s + R_V)}{g^2} (2.9 \times 10^{10}) \right]^{1/2} \quad (183)$$

It is evident that even when T_r is $1000^\circ K$, it is negligible compared to the term containing λ^6 . Assuming the bandwidth of the circuits following the amplifier is 3 kc, equation (183) now becomes

$$E_{min} = 1.16 \times 10^{-5} \sqrt{R_s + \frac{R_V}{g^2}} \frac{\text{volts}}{\text{meter}} \quad (184)$$

or

$$E_{min} = 11.6 \sqrt{R_s + \frac{R_V}{g^2}} \frac{\text{microvolts}}{\text{meter}} \quad (185)$$

It is apparent that it is desired to reduce the term R_V/g^2 by reducing the equivalent noise input resistance or by increasing the gain. For one set of experimental amplifiers constructed from 2N2218 transistors for the 8-loop spaced loop described above, the following measurements were made:

1. Amplifier voltage gain = 22 at 3 megacycles.
2. A signal input of $.11 \times 10^{-6}$ volts produces a 10-db signal plus noise-to-noise ratio at the output for a 3-kc bandwidth.

Converting the 10 db to a voltage ratio of 3.16 we have

$$\frac{\text{Signal} + \text{Noise}}{\text{Noise}} = 3.16$$

where signal = $.11 \times 10^{-6}$ volts. Therefore, the noise is given by

$$\begin{aligned} 2.16 \text{ Noise} &= .11 \times 10^{-6} \\ \text{Noise} &= .0509 \times 10^{-6} \text{ volts} \end{aligned}$$

The noise output is therefore $(22)(.0509 \times 10^{-6})$ volts or 1.12 microvolts. Therefore, the equivalent noise input resistance is related to the measured output by

$$e^2 = 4kT_v R_v B$$

or

$$1.25 \times 10^{-12} = (4)(1.374 \times 10^{-23})(3 \times 10^3) T_v R_v$$

for the assumed temperature of $T_v = 290^\circ \text{K}$

$$R_v = 2.61 \times 10^4 \text{ ohms for 3 kc bandwidth}$$

Substitution of this value, the gain and a loss resistance of 3 ohms into equation (185) yields

$$E_{\min} = 11.6 \sqrt{3 + \frac{2.61 \times 10^4}{484}} = 87.5 \frac{\text{microvolts}}{\text{meter}} \quad (186)$$

This is obtained for a one to one signal-to-noise ratio; for a 10-db signal-to-noise ratio, this converts to

$$E_{\min} = 189 \mu\text{volts/meter}$$

The experimental value reported in the interim report for this contract dated 1 February 1963 corresponds to the conditions assumed above and was 175 μvolts per meter at 3 megacycles.

b. A Tuned Spaced Loop with a Transistor Amplifier

Equation (181) is equally applicable to any case where the various noise resistances and their temperature can be specified. When the antenna is tuned, voltage gain is applied to the antenna resistance noise voltages before the amplifier. Therefore the resistances presented to the input of the amplifier will be amplified by the gain squared of the tuning network or Q^2 at resonance. Thus at the output of the amplifier the signal-to-noise ratio is one to one when equation (181) has the form

$$E_{\min} = \frac{16\pi \sqrt{2kB}}{\lambda} \left[T_r + \frac{(Q^2 g^2 R_s T_s + T_v R_v) \lambda^6}{Q^2 g^2 2.46 \times 10^5 (A_r d)^2} \right]^{1/2} \quad (187)$$

It should be noted that the improvement which can be obtained by tuning is not as great as might be expected because the value of Q will be limited by the input resistance R_{in} of the amplifier. Since R_{in} is contained in R_v these parameters will be interrelated, in addition the gain of the amplifier may be affected by the increased source resistance. For a practical case analogous to the one given above in Section a, however, the gain could be expected to be on the order of 20 with the antenna tuned with a Q of 10. Substitution of these values into equation (187) using the same values as in the previous example for the other parameter yields

$$E_{\min} = 1.16 \times 10^{-5} \sqrt{R_s + \frac{R_v}{(Qg)^2}} \quad \frac{\text{volts}}{\text{meter}} \quad (188)$$

For a loss resistance of 3 ohms, a Q of 10, a gain of 20 and $R_v = 2.61 \times 10^4$ equation (188) yields

$$E_{\min} = 11.6 \sqrt{3 + \frac{2.61 \times 10^4}{4 \times 10^4}} = 22.2 \frac{\mu\text{volts}}{\text{meter}} \quad (189)$$

This is for a one to one signal-to-noise ratio. For a 10-db signal-to-noise ratio this converts to

$$E_{\min} = 48.0 \frac{\text{microvolts}}{\text{meter}}$$

This represents an improvement over the aperiodic case of about four to one. This improvement is obtained at the inconvenience of direct tuning the antenna. The inconvenience in this case may be considerable, however, as it is not possible to easily direct tune a rotating

antenna, hence the antenna must be arranged into two channels (say North-South and East-West) which are phase and gain matched. The improvement obtained in this case may not be worth the difficulty encountered in circuit matching.

c. An Indirectly Tuned Spaced Loop with a Transistor Amplifier

In this case a rotating antenna may be considered which is transformer coupled to the amplifier. The secondary of the transformer is stationary and is tuned with a Q of 10. If the transformer introduces a loss resistance of 3 ohms and has a coupling coefficient of .8, other factors remaining the same as in example B, equation (181) becomes

$$E_{\min} = 11.6 \sqrt{6 + \frac{2.61 \times 10^4}{[(10)(.8)(20)]^2}} = 30.8 \frac{\text{microvolts}}{\text{meter}} \quad (190)$$

For a 10-db signal-to-noise ratio this becomes

$$E_{\min} = 66.5 \frac{\text{microvolts}}{\text{meter}}$$

This value is roughly a three to one improvement over the aperiodic case but is somewhat poorer than the direct tuned case.

Bond²⁴ has calculated examples for a simple loop which are similar to the above three examples except that he provides detail relating the loss resistances to the assumed Q's. His results also show that the direct tuned case provides maximum sensitivity, the indirect tuned case somewhat lower sensitivity. He does not make a direct comparison with the aperiodic case; however it is evident that this will be the lowest sensitivity case of the three considered.

In summary it is to be noted that in general the equivalent noise input resistance and temperature of the first amplifier is generally the limiting factor. However, as gain is increased, either in or preceding the first

24. Bond, Donald S., Radio Direction Finders, McGraw-Hill Book Company, Inc., New York, 1944.

amplifier, the loss resistances of the antenna and coupling networks become the next limiting factor, provided the equivalent noise input resistance (and temperature) of the amplifier remains at a reasonable value as gain increases. Therefore, the applicability of cooling to reduce resistance both in the amplifier and the antenna and coupling networks becomes apparent. The ultimate limit on sensitivity (if such techniques could be carried to an extreme) would be the noise temperature of the environment of the antenna; however, for a low radiation resistance antenna this seems to be a remote goal even using cryogenic techniques.

17. Impedance of the Spaced Loop Antenna

Calculations have already been given for the radiation resistance of the spaced loop. The reactance portion of the impedance may be determined from an analysis by Burgess²⁵. This analysis provides the input impedance of a lossless screened loop based on the assumption that the uniform distributed parameter transmission line equations apply. By this method the impedance appearing at the terminals of a balanced screened loop with an open circuited gap at low frequencies is given by

$$Z = j\omega L \quad (191)$$

where L is the low frequency inductance. At higher frequencies Burgess shows that the loop is first resonant when the loop perimeter is a half wavelength and is first antiresonant when the loop perimeter is between $\lambda/4$ and $\lambda/4.06$.

When two loops are combined in a parallel connected spaced loop, the first resonance and first antiresonance will occur at lower frequencies than those just given because of the transmission line action of the crossover arms. At the crossover the impedance will appear to be that of two transmission lines in parallel, each terminated in an impedance corresponding to the loop impedance. Thus, the impedance at the crossover is

$$Z_{SL} = Z_o \frac{Z_L \cos \beta r_d + j Z_o \sin \beta r_d}{Z_o \cos \beta r_d + j Z_L \sin \beta r_d} \quad (192)$$

where Z_L is one-half one loop impedance and Z_o is one-half the characteristic impedance of one arm. At frequencies below the first antiresonance (that is at $\lambda > 4p$ where p is the loop perimeter) the impedance then is:

25. Burgess, R. E., "Screened Loop Aerials," Wireless Engineer, May 1944, p. 210.

$$Z_{SL} = jZ_0 \frac{\omega L \cos \beta r_d + Z_0 \sin \beta r_d}{Z_0 \cos \beta r_d - \omega L \sin \beta r_d} \quad (193)$$

This becomes antiresonant when $Z_0 \cos \beta r_d = \omega L \sin \beta r_d$ or when

$$\tan \beta r_d = \frac{Z_0}{\omega L} \quad (194)$$

For an 8-loop array with four loops in each channel combined at the crossover point, the characteristic impedance will be one-fourth that of one arm of the crossover, and Z_L will be one-fourth the impedance of one loop.

It is evident that when $Z_0/\omega L$ is in the vicinity of unity, an anti-resonance occurs at βr_d near 45° , and this will occur at a frequency which is lower than that corresponding to $\lambda = 4p$. If the loops are small and Z_0 made large, the antiresonance will still occur below $\beta r_d = \pi/2$ or $r_d = \lambda/4$. Therefore, to operate a spaced loop with a spacing of 17 feet (as in the 8-loop array now under investigation) one must tolerate an antiresonance below $f = 29$ megacycles. It was reported in the final report dated 30 September 1962 that this experimental antenna has its first antiresonance near 12.5 megacycles. In other words the antenna will always be anti-resonant below the frequency where the arms are a quarter wave long, and when the loop inductance is not negligible, the resonance will be considerably lower than this frequency.

The above data can be used to calculate the inductance of the experimental loops, for at 12.5 mc equation (194) is approximately valid if the loops are not very large and

$$\tan \left[\frac{(2\pi)(12.5)(17)(.305)}{(300)(2)} \right] = \frac{Z_0}{L} \left[\frac{1}{(2\pi)(12.5)(106)} \right]$$

or

$$Z_0 = 6.35 \times 10^7 L$$

If $Z_0 = 100$ ohms, $L = 1.5 \times 10^{-6}$ henry, which is a typical value for a loop of this size. Although Z_0 is unknown, it is probably not less than 25 ohms for 4 arms so that the single loop antenna inductance is at least 1.5 microhenries. Note the resonance occurred when r_d was less than one-eighth wavelength long for a reasonable value of L . This is typical of this type of spaced loop, and one may adopt a rule of thumb that when the loops have a

diameter which is less than half the spacing but not small compared to the spacing the first antiresonance will be somewhere in the vicinity of $\lambda/8$. If the loops become large, this rule will fail and the resonance will be still lower.

It should also be evident that the first resonance (which will occur above the antiresonance) will also occur at a lower frequency than for a single simple loop. It will occur at a frequency which is less than that for which $r_d = \lambda/2$ for any loop impedance, but in general it will occur at a frequency less than twice the first antiresonance frequency. Thus for the 8-loop spaced loop previously described, the first resonance should occur at less than 25 mc. Experimentally it was observed near 21 mc.

The antenna impedance may be calculated in greater detail by reference to Burgess²⁶ paper and applying the formulas given there to the crossover arm transmission lines. These formulas are more accurate than those given here; however, the procedures involved for determining the loop inductance coupling to shield and capacitance are lengthy. The formulas given above are sufficient as a guide for approximately determining resonance and antiresonance of the spaced loop.

18. Other Spaced Loop Characteristics

a. Reradiation Error Reduction

The ability of the coaxial spaced loop antenna to perform as a direction finder in the presence of reradiation with less error than a simple loop is now well known²⁷, and is the primary reason for using the antenna in high frequency shipboard direction finding. The general spaced loop analyzed herein does not yield to the same simple analysis as was used to show the coaxial spaced loop error reduction. For instance in the final report of contract NObsr-64585, it was shown that when both the target transmitter and the reradiator are in the far field of the D/F antenna, the response of the D/F antenna is

$$E = f(\theta - \delta) + Ae^{jT} f(\theta) \quad (195)$$

26. Burgess, R. E., "Screened Loop Aerials," Wireless Engineer, May 1944, p. 210-221.

27. Travers, D. N., et al., "Methods for the Reduction of Reradiation Error in Naval High Frequency Shipboard Direction Finding," Final Development Report, Southwest Research Institute, 1 January 1961.

where $f(\theta - \delta)$ is the azimuth pattern response to the target transmitter, θ is the azimuth of the rotating D/F antenna at any instant, δ is the azimuth of the target transmitter relative to the reradiator, Ae^{jT} is the reradiated field relative to the incident field and $f(\theta)$ is the azimuth pattern response of the D/F antenna to the reradiator. In the notation of the present report equation (195) would be rewritten as

$$E = f_a(\phi - \delta) + Ae^{jT} f_b(\phi) \quad (196)$$

where ϕ is now the azimuth angle and the functions f_a and f_b are allowed to be different. Equation (42) shows that $f_a = f_b$ for a reradiator at any distance reasonably greater than r_d , for a coaxial spaced loop. The observed bearing ϕ_R and error ϵ formulas can be derived from equation (196) and are

$$\tan 4\phi_R = \frac{\sin 4\delta + 2A \sin 2\delta \cos T}{\cos 4\delta + 2A \cos 2\delta \cos T + A^2} \quad (197)$$

$$\tan 4\epsilon = \frac{-2A \sin 2\delta \cos T - A^2 \sin 4\delta}{1 + 2A \cos 2\delta \cos T + A^2 \cos 4\delta} \quad (198)$$

where δ = true azimuth of the wanted signal if the reradiator is at zero azimuth.

The reradiation error analysis was given in detail in previous reports where it was shown that one may also define a blurring parameter ϕ_I given by²⁸

$$\tanh 4\phi_I = \frac{-2A \sin 2\delta \sin T}{1 + 2A \cos 2\delta \cos T + A^2} \quad (199)$$

All these formulas show a two to one improvement over the simple loop in error performance. In addition multivalued observed bearing

28. Travers, D. N., et al., "A Spinning Multiloop Direction Finder for the Reduction of Shipboard Reradiation Error," Task Summary Report Number IV, Southwest Research Institute, 1 September 1958, page 7.

data (re-entrants) do not occur for the spaced loop for thin reradiators. This was shown mathematically²⁹ and experimentally³⁰.

b. Radial Nulls

From the general field equations (37), (38) and (39), it is apparent that it is incorrect to assume near and far field patterns are similar. It is therefore true that patterns for arrays involving spaced loops with other modes such as the simple loop, the near and far field patterns may be different. An example of this has been reported for the case of the 3-loop array consisting of a coaxial spaced loop and a sense loop.

The pattern variation with distance suggests that arrays could be designed having "radial nulls" that is, zero response at a certain distance from the antenna in a certain direction and nonzero response at other distances in the same direction. This corresponds to a local cancellation of two dissimilar induction fields.

As an example consider the coplanar spaced loop with a parallel sense loop and may be studied by combining equations (42) and (54). This yields for vertical polarization

$$E_{\theta} = \left\{ \frac{-j(I_m L)\beta^3 r_d}{2\pi} \sin \theta \left[\left(\frac{1}{(j\beta r)^3} + \frac{1}{(j\beta r)^2} \right) (\cos 2\phi - \sin^2 \phi) - \frac{1}{j\beta r} \sin^2 \phi \right] g + \frac{\beta^2 (I_m L)_3 \sin \phi}{4\pi} \left[\frac{1}{(j\beta r)^2} + \frac{1}{j\beta r} \right] \right\} e^{j\omega \left(t - \frac{r}{c} \right)} \quad (200)$$

where g is an arbitrary constant.

-
29. Travers, D. N., et al., "A Spinning Multiloop Direction Finder for the Reduction of Shipboard Reradiation Error," Task Summary Report Number IV, Southwest Research Institute, 1 September 1958, p. 12.
30. Moore, J. D., et al., "Shipboard Test of the Final Engineering Model of the Three-Loop Antenna on the U.S.S. Richard E. Kraus," Interim Report, Southwest Research Institute, 26 August 1960, Table 9, p. 56.

It is evident that if g is chosen to produce a null in the far field pattern, at say $\phi = \pi/2$, a similar null does not occur in the near field pattern. Similarly if g is chosen to produce a null in the near field, a nonzero response occurs in the far field for the same direction. The latter condition leads to some interesting conclusions.

For instance if we let r approach zero, equation (200) becomes

$$E_{\theta} = \frac{\beta^2 e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi} \left[-g \frac{j(I_m L)_1 \beta r_d}{(j\beta r)^3} (\cos 2\phi - \sin^2 \phi) \sin \theta + \frac{(I_m L)_3 \sin \phi}{2(j\beta r)^2} \right] \quad (201)$$

Substituting from equation (5) yields

$$E_{\theta} = \frac{\beta^2 e^{j\omega \left(t - \frac{r}{c} \right)}}{2\pi(j\beta r)^2} \left[-g \frac{\omega \mu I_1 A_1 \beta r_d}{j\beta r} \sin \theta (\cos 2\phi - \sin^2 \phi) - \frac{j\omega \mu I_3 A_3 \sin \phi}{2} \right] \quad (202)$$

Let $\theta = \pi/2$, $\phi = \pi/2$ corresponding to horizontal incidence along the plane of the loops. Also let $I_1 = I_3$ and $A_1 = A_3$, then

$$E_{\theta} = \frac{-j\beta^2 \omega \mu I A}{2\pi(j\beta r)^2} e^{j\omega \left(t - \frac{r}{c} \right)} \left[\frac{2gr_d}{r} + \frac{1}{2} \right] \quad (203)$$

The E_{θ} field is zero at a distance $r = 4gr_d$. When $g = 1$ and $r \gg \lambda$ the response is

$$E_{\theta} = \frac{j\omega \mu I A \beta}{2\pi j\beta r} e^{j\omega \left(t - \frac{r}{c} \right)} \left[j\beta r_d - \frac{1}{2} \right] \quad (204)$$

The term $j\beta r_d$ is negligible with respect to $1/2$ when

$$|j\beta r_d| \ll 3\% \text{ of } 1/2$$

Therefore

$$\frac{2\pi r_d}{\lambda} = .015, \text{ or } r_d = .00239\lambda$$

At 30 mcs $\lambda = 1000$ cm and $r_d = 2.39$ cr. or about one inch. This requires for $g = 1$ that $r = 9.56$ cm.

In summary, for a location very close to the near field source (reradiator) a zero response to the reradiator is theoretically obtained, but the far field loop pattern is changed no more than 3 percent. This suggests the possibility of a crossed simple loop in the near vicinity of a reradiator without appreciable error, however, in the derivation of the general field equations it was assumed that r_d^2 was negligible with respect to r^2 . Therefore, the above analysis showing a radial null at $r = 4gr_d$ can only be approximate. An experimental approach appears to be more useful than a repeat of the analysis with the $r_d^2 \ll r^2$ restriction removed, however, time has not as yet been available for such investigation. Experimentally it would be more practical to attempt to adjust r for a given g to obtain $E_\theta = 0$ for one distance. Other radial null effects can be found by further investigation of the equations.

c. Difficulty of Constructing the Spaced Loop

It is well known that loop alignment and balance are major problems in the spaced loop antenna relative to analogous problems for the simple loop antenna. In designing a small spaced loop ($Ar_d \ll \lambda^3$) pattern distortion will be observed to increase very rapidly with decreasing frequency. This condition is evidently impossible to avoid because it is not possible to make the loops exactly alike, exactly parallel, or to install a perfect difference connection at the crossover point of the loop input leads. The imperfections have dipole equivalent circuits, and the fields of these dipoles do not decrease in amplitude as fast as the spaced loop (quadrupole) fields. This means that as frequency is decreased a point is reached where dipole distortion is greater than some acceptable level.

An analysis of spaced loop patterns to determine quantitatively the extent of dipole and omnidirectional distortion in a measured pattern has been reported³¹. Experimental results have been reported in a number of interim reports for contract NObsr-64585. In general

31. Travers, D. N., et al. Interim Development Report, Southwest Research Institute, 26 October 1957, p. 19.

it has been found that small spaced loops may be constructed for use over the entire high frequency range if sufficient care is used in the loop alignment and installation of crossover leads. Crossover lead installation is generally more important (or difficult to achieve) than loop alignment.

The smallest air core spaced loop for HF use successfully tested with an undistorted pattern at 3 mc had a size $Ar_d = 250 \text{ in}^3$. In this case r_d was about 8 inches. Smaller ferrite spaced loops have been constructed with equal performance.

In general the smallest spaced loop which can be constructed for use at a given frequency is not limited by difficulty of construction, however, for it has been found possible to construct spaced loops which are so small as to be of little use because of low sensitivity.

19. Comparison with Equivalent Factors for the Simple Loop

a. Effective Height of a Simple Loop

By an analysis similar to the one previously given for the spaced loop, the effective height of a simple loop in free space is

$$h_e = \frac{2\pi NA}{\lambda} = \beta NA \quad (205)$$

or if a core with a permeability of μ_r is inserted through the loop, then the effective height is

$$h_e = \frac{2\pi NA\mu_r}{\lambda} = \beta NA\mu_r \quad (206)$$

b. Radiation Resistance of a Simple Loop

The radiation resistance of a simple loop in free space is given by Kraus³² as

$$R_r = 20\beta^4 A^2 N^2 = 3.12 \times 10^4 \left(\frac{NA}{\lambda^2} \right)^2 \text{ ohms} \quad (207)$$

32. Kraus, J. D., Antennas, McGraw-Hill Book Company, New York, 1950, p. 167.

c. Gain of the Simple Loop

The gain of the simple loop is found by substituting equations (205) and (207) into equation (143) thus

$$g = 1.5 \quad (208)$$

d. Effective Area of Simple Loop

The effective area is given by equation (129) hence

$$A_e = \frac{1.5\lambda^2}{4\pi} \quad (209)$$

e. Signal-to-Noise Ratio of Simple Loop

Equations (174) and (176) give the signal-to-noise ratio for an antenna. Substituting equations (205) and (207) into equation (176) yields the signal-to-noise ratio

$$P = \frac{n_a E \beta A \lambda^2}{2 \sqrt{kB} [T_r 3.12 \times 10^4 A^2 n^2 + T(\Sigma n_s^2 R_s + R_v) \lambda^4]^{1/2}} \quad (210)$$

f. Minimum Observable Field Strengths

The minimum field strength is given by equation (179). Substitution of values for h_e and R_r [equations (205) and (207)] yields for the perfect loop

$$E_{(o)min} = \frac{\sqrt{4kBT_r R_r}}{h_e} = 8.94 \beta \sqrt{kBT_r} \quad (211)$$

This value is not significantly different from that of the spaced loop. The minimum field for the practical loop is given by equation (179) with appropriate substitutions or

$$E_{min} = 8.49 \beta \sqrt{kBT_r} \left[1 + \frac{T}{T_r} \left(\frac{\Sigma n_s^2 R_s + R_v}{20 \beta^4 A^2 n_a^2 N^2} \right) \right]^{1/2} \quad (212)$$

The ratio of the simple loop minimum field to the coaxial spaced loop (parallel connected), is the ratio of equation (212) to (181), or when R_s , R_v , T , T_r , B , and $N = 1$ are the same in both cases and $1 \ll T(\Sigma n_s^2 R_s + R_v) / T_r(20\beta^4 A^2 n_a N^2)$ this ratio is

$$K_g = \frac{E_{\min}(\text{loop})}{E_{\min}(\text{sp. loop})} = \frac{\pi r_d}{\lambda} \quad (213)$$

which is also the ratio of the effective heights. Thus at 3 mc and $r_d = 1$ meter, the simple loop is 31.4 times above the spaced loop or approximately 30 db more sensitive.

20. Conclusions

a. By means of a straightforward procedure using retarded potentials, a set of general field equations have been derived for a general spaced loop. The coaxial, vertical coplanar and horizontal coplanar spaced loops may be treated as special cases and response patterns determined for vertical and horizontal polarization in both azimuth and elevation planes. In general the equations show that for certain cases the shape of the field pattern is not the same in the near field as in the far field.

b. Important special cases in which the field pattern shape changes as a function of distance are vertical polarization (E_θ and H_ϕ) in the azimuth plane for the vertical coplanar spaced loop and horizontal polarization (E_ϕ and H_θ) in the elevation plane for the horizontal coplanar spaced loop. The general spaced loop, formed when the loops are parallel but are neither coaxial nor coplanar, will have at least one field component which changes shape as a function of distance to the source.

c. The coaxial spaced loop is the only special case where none of the electric or magnetic field components change as a function of distance to the source.

d. In those cases where the pattern changes as a function of source distance, the shape change produced for any given polarization depends on whether the electric field or the magnetic field is observed. For instance, for the vertical coplanar spaced loop for vertical polarization, the far field patterns for E_θ and H_ϕ are equal while the near field patterns are not equal.

e. The simple loop field pattern shape is not a function of distance to the source as long as this distance is greater than the loop dimensions.

f. Certain combinations of spaced loops and simple loops are useful to produce sense patterns which remove the inherent ambiguities in the spaced loop patterns. For instance the coaxial spaced loop for vertical polarization has a four-way ambiguity which may be resolved with a single simple loop if the antenna is rotated. Such sense patterns will be most useful when formed by combinations which do not exhibit pattern shape changes as a function of distance to the source.

g. Any spaced loop will respond differently to nearby reradiators than to far field sources. In certain special cases these response differences will include field pattern shape changes as a function of distance to the source.

h. The effective height, radiation resistance, gain, effective area, signal-to-noise ratio, and minimum observable field strength are all parameters which may be derived directly from the field equations and Nyquist's theorem concerning thermal noise. In general it is found that when the spaced loop dimensions are small compared to the wavelength, the effective height is small and the radiation resistance is small. For practical sized loops in the vicinity of 3 mc, the radiation resistance may be as low as 10^{-8} ohms which is completely negligible compared to the loss resistance in the antenna. The gain and effective area, however, have values which are not significantly different from that of a half-wave dipole.

i. The effective height of the coplanar spaced loop is twice that of the coaxial spaced loop. The effective height of a series connected pair of loops forming the spaced loop is also twice that of parallel connected loops.

j. The radiation resistance of the coplanar spaced loop will be twice that of the coaxial spaced loop. In either case the radiation resistance of the series connection will be four times that of the parallel connection.

k. The maximum gain of the vertical coplanar spaced loop for vertical polarization will be twice that of the coaxial spaced loop. The effective area of the vertical coplanar spaced loop will be twice that of the coaxial spaced loop for the lossless case.

l. The signal-to-noise ratio of the general spaced loop, when the dimensions are small compared to the wavelength so that the radiation resistance is less than say a tenth ohm, will be determined by the equivalent noise input resistance and temperature of the first amplifier if estimates are based on conventional modern design techniques. This leads to the conclusion that the equivalent noise input resistance or temperature of the first amplifier must be reduced, or gain at near unity noise figure must be introduced ahead of the first amplifier. Theoretically this can be accomplished by parallel tuning the antenna to obtain a gain equal to the Q of the tuned circuit. Calculations show this will provide a significant improvement in a signal-to-noise ratio or reduction of minimum observable field strength. This

technique will be limited by equivalent noise resistance and temperature of the antenna (apart from the radiation resistance and noise temperature of the environment). Theoretically this noise source (that due to the loss resistance) may be reduced by cryogenic techniques thus improving the signal-to-noise ratio still further. However, an ultimate limit is reached which is determined by the radiation resistance of the antenna and the temperature ascribed thereto which is the equivalent noise temperature of the environment of the antenna, that is, the earth, the sun, etc.

m. The signal-to-noise ratio, which would exist if all noise sources other than the radiation resistance considered at the environmental temperature of the antenna were eliminated, is estimated to be far in excess of that which the present state of the art will permit to be observed. This is primarily because of the typical equivalent noise input resistance of the best available amplifiers.

n. This ultimate signal-to-noise ratio is determined only by the noise temperature of the environment of the antenna and is independent of the size of the spaced loop. For a frequency of 3 mc and a temperature of 290°K and a bandwidth of 10 kc, the minimum observable field strength determined by the noise temperature of the antenna is in the vicinity of .002 to .004 microvolts per meter depending on the type of spaced loop. It is theoretically possible to observe such field strengths with spaced loops only if all loss resistance is eliminated, the noise contributed by the first amplifier is made negligible, and the noise temperature of the antenna is in the vicinity of 290° or less.

o. Calculations indicate that the present state of the art is such that a reasonable sized spaced loop parallel tuned with a low noise transistor amplifier could observe a minimum field strength in the vicinity of 50 microvolts per meter at 3 mc with a 10-db signal-to-noise ratio.

p. At low frequencies, the impedance of the spaced loop is inductive. As the frequency is increased, the impedance appears to be that of a transmission line terminated in an inductance. This transmission line corresponds to the support arms of the spaced loop. The first antiresonance (the lowest nonzero frequency of zero reactance) will occur for reasonable sized loops when the length of the support arm (distance from the center of the spaced loop to one loop) is on the order of one-eighth wavelength. With appropriate design procedures this antiresonance could be moved to a high frequency, but under no circumstances could it be made to clearly approach an arm length of one quarter wavelength. The first resonance will occur under ordinary conditions when the arms are about one quarter wavelength long (or when the array is about one-half a wavelength in diameter or less).

q. Reradiation error reduction has been covered in other reports and for null type direction finding appears to be most practically achieved with the coaxial spaced loop.

r. It is theoretically possible to obtain a null response in a given direction at a specific distance and only at a specific distance from the antenna as the result of the fact that certain patterns change shape with distance to the source. These so-called radial nulls could be obtained by matching a simple loop to a spaced loop for a source at a specific distance.

21. Bibliography of Spaced Loop Reports and Papers

1. Anonymous, "Radio Direction Finding, " War Department, War Department Technical Manual TM 11-476; July 1947.
2. Anonymous, "Trials of High Frequency Direction Finder Outfit RH2X in H. M. S. Saltburn and H. M. S. Queen Elizabeth, " (CONFIDENTIAL). Report M 756, Admiralty Signal Establishment; December 1945.
3. Bond, D. S., "Radio Direction Finders, " McGraw-Hill Book Co., Inc., New York, N. Y.; 1944.
4. Caplin, F., and Bagley, J. H., "A Mobile Spaced Loop Direction Finder, " Journal of the Institution of Electrical Engineers, Vol. 94, Part IIIA, pp. 676-682; 1947.
5. Chu, L. J., "First Report on High Frequency Direction Finder Problems, " (CONFIDENTIAL), First Report on Item No. 6 of Contract No. NObsr 57301, Index No. NE-121021, Subtask 15, Naval Electronics Laboratory; 20 November 1953.
6. Chu, L. J., "Second Report on High Frequency Direction Finder Study, " (CONFIDENTIAL), Second Report on Item No. 7 of Contract No. NObsr 57301, Index No. NE-121021, Subtask 15, 15 March 1954.
7. Chu, L. J., "Third Report on High Frequency Direction Finder Study, " (CONFIDENTIAL), Report No. 8 on Item No. 8 of Contract No. NObsr 57301, Index No. NE-121021, Subtask 15, 21 June 1954.
8. Crampton, C., "A Note on the Application of Spaced Loop High Frequency Direction Finder in H. M. Ships, " Report M 433, Direction Finder Section, Admiralty Signal Establishment; July 1942.
9. Crampton, C., "Naval Radio Direction Finding, " Journal of the Institution of Electrical Engineers, Vol. 94, Part IIIA, No. 15, pp. 132-153; 1947.

10. Crampton, C., Struszynski, W., de Walden, S., and Redgment, P. G., "Some Principles Underlying the Design of Aerial Systems for High Frequency Radio Direction Finders in H.M. Ships," Journal of the Institution of Electrical Engineers, Vol. 95, Part III, pp. 437-453; 1948.
11. Evans, G., "The Crossed-Spaced-Loop Direction-Finder Aerial," IRE Transactions on Antennas and Propagation, Vol. AP-10, No. 6, pp. 686-691; November 1962.
12. Friis, H. T., "A New Directional Receiving System," Proceedings IRE, Vol. 13, pp. 685-707; December 1925.
13. Horner, F., "An Experimental Spaced Loop Direction Finder for Very High Frequencies," Journal of the Institution of Electrical Engineers, Vol. 94, Part III, No. 28, pp. 126-133; March 1947.
14. Keen, R., "Wireless Direction Finding," Fourth Edition, Iliffe and Sons, Ltd., London, England; 1947.
15. Keen, R., Bray, W. J., Smith, S. B., Redgment, P. G., Bankes, E. W. J., Horton, C. E., Stansfield, R. G., Ross, W., and Crampton, C., "Discussion on 'Direction Finding' at the Radio Communication Convention," Journal of the Institution of Electrical Engineers, Vol. 94, Part IIIA, No. 15, pp. 867-870; 27 March 1947.
16. Moore, J. D., et al., "Shipboard Test of the Final Engineering Model of the Three Loop Antenna on the USS Richard E. Kraus," Interim Development Report, Southwest Research Institute; 26 August 1960.
17. Moore, J. D., et al., "The Design and Construction of a Three-Loop Antenna for High Frequency Shipboard Direction Finding," Task Summary Report Number VI, Southwest Research Institute; 26 October 1960.
18. Moore, J. D., et al., "Progress in the Development of a Twin Channel Spaced Loop Direction Finder," Interim Development Report, Southwest Research Institute; 1 May 1962.

19. Moore, J. D., et al., "Progress in the Development of a Direction Finding System Utilizing the 8-Loop Fixed Spaced Loop Antenna," Interim Development Report, Southwest Research Institute; 1 August 1962.
20. Moore, J. D., et al., "Progress in the Development of a Direction Finding System Utilizing the 8-Loop Fixed Spaced Loop Antenna," Final Development Report, Southwest Research Institute; 30 September 1962.
21. Ross, W., "The Development and Study of a Practical Spaced Loop Radio Direction Finder for High Frequencies," Journal of the Institution of Electrical Engineers, Vol. 94, Part III, No. 28, pp. 99-107; March 1947.
22. Schlicke, Dr. Heinz, "Electronics Research in the German Navy, Series of Lectures Delivered by Freg. Kapt. Dr. Ing. Schlicke," (CONFIDENTIAL), Division of Naval Intelligence; 15 September 1945.
23. Seeley, Elwin W., "Two- and Three-Loop Superdirective Receiving Antennas," Journal of Research of the National Bureau of Standards - D. Radio Propagation, Vol. 67D, No. 2, March-April 1963, pp. 215-235.
24. Sherrill, W. M., et al., "Progress in the Development of a Twin Channel Spaced Loop Direction Finder and the Evaluation of the Racal RA. 17 and R-390A Receivers," Interim Development Report, Southwest Research Institute; 1 February 1962.
25. Terman, Frederick E., "Radio Engineers' Handbook," McGraw-Hill Book Co., Inc., New York, N. Y.; 1943.
26. Travers, D. N., et al., "A Spinning Multiloop Direction Finder for the Reduction of Shipboard Reradiation Error," Task Summary Report Number IV, Southwest Research Institute; 1 September 1958.
27. Travers, D. N., et al., "Experiments on a Three Loop Antenna Electrostatic Screen, and Rotary Printed Circuit Transformer," Interim Development Report, Southwest Research Institute; 26 October 1958.

28. Travers, D. N., et al., "Progress on the Design and Construction of a Three Loop Prototype Antenna for Use with an AN/SRD-7 Receiver," Interim Development Report, Southwest Research Institute; 26 January 1959.
29. Travers, D. N., et al., "Interim Progress on the Construction of a Three Loop Prototype Antenna," Interim Development Report, Southwest Research Institute; 26 April 1959.
30. Travers, D. N., et al., "Assembly of the Three Loop Antenna and an Investigation of Improved Methods for Reducing Reradiation Error," Interim Development Report, Southwest Research Institute; 26 July 1959.
31. Travers, D. N., et al., "Final Preshipboard Testing of the Three Loop Direction Finding Antenna," Interim Development Report, Southwest Research Institute; 26 October 1959.
32. Travers, D. N., et al., "A Listing of Engineering Tolerances for a Rotating High Frequency Three Loop Antenna," Task Summary Report Number VII, Southwest Research Institute; 25 November 1960.
33. Travers, D. N., et al., "Methods for the Reduction of Reradiation Error in Naval High Frequency Shipboard Direction Finding," Final Development Report, Southwest Research Institute; 1 January 1961.
34. Travers, D. N., et al., "A Method of Goniometer Scanning the Coaxial Spaced Loop Direction Finder for Vertical Polarization," Task Summary Report Number VIII, Southwest Research Institute; 1 April 1961.
35. Whipple, R. T. P., "Preliminary Theoretical Report on the Errors Caused in Bearings by Reradiation from a Vertical Conductor Placed Near a Spaced Loop Direction Finder," Report M. 585, Admiralty Signal Establishment; December 1943.

22. Activities for the Interim Period

In addition to the theory of the spaced loop antenna reported herein and mention of experimental data applicable thereto, activities during the period have been concentrated on the preparation of an 8-loop direction finding system with simultaneous twin channel and goniometer displays. This system was described in the previous interim report dated 1 February 1963 and is outlined in Figure 5 of that report.

The arrangement reported at that time is now substantially complete and ready for tests. The third channel receiver (Racal RA. 117) has been received and partially slaved to the twin channel receiver. Preliminary investigation shows that the three channel arrangement is suitable for sensing but the phase tracking of the third channel is not as close as the tracking between channels of the original twin receiver. Amplitude matching of the third channel is not required for the present system.

Testing of this system will begin within the next few weeks and will continue throughout the next interim period. A detailed reporting will be made at a later time. It is recommended that a representative of the Bureau of Ships visit Southwest Research Institute during the next interim period and observe this system in operation with the mast top 8-loop antenna.

In addition to preparation of the D/F system described above, further work has been performed on the investigation of throughmast antennas. This work is continuing, and a detailed reporting will be made at a later date.

Collection of information applicable to the more advanced version of the twin channel and goniometer 8-loop D/F system has continued with emphasis on availability of frequency synthesizers and storage tube CRT indicators.

Testing of the DFG-4C UHF direction finder at Newport, Rhode Island, was completed for two nonoptimum sites on a destroyer. The results of this test have been summarized in a report which was very recently submitted. Further preparations for the shipboard tests of the 3-loop antenna in a nonoptimum site have continued. Present planning is for the test to be conducted at three heights on an aftmast location in May of this year.